EIGEN-SOLUTION FOR SHANNON ENTROPY WITH TRIGONOMETRIC YUKAWA PLUS INVERSELY QUADRATIC POTENTIAL.

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ABSTRACT
Shannon entropic measures for a considerable potential provides a clear understanding of some quantum mechanical systems in direct application to quantum information and signal processing. In this work, we have provided an analytical eigen -solution for Shannon entropy for trigonometric-Yukawa plus inversely quadratic potential (TYPIQP). The wave function used for the Shannon eigen solution is obtain through bound state solution of Schrodinger equation with the novel potential via Nikiforov-Uvarov method with the help of Greene-Aldrich approximation to the centrifugal term. The normalized wave function is obtained through the combination of two infinite series of confluent hypergeometric functions. The numerical bound state solutions were obtained for a particular quantum state but for different orbital angular quantum number using MATLAB programme for different values of real constant adjustable parameter $\alpha = 0.01, 0.02, 0.03, 0.04 \, and \; 0.05$. The results show that the numerical bound state energies increase with an increase in quantum state, while the energy spectral diagrams show unique quantization of different quantum state. The complex part of the Shannon eigen solution were carried out using MATHEMATICA.

INTRODUCTION
Quantum information has a lot of application in the field of engineering, physical sciences and applied mathematics. Entropic measure plays a significant role in information theory especially as it provides clear understanding of some quantum mechanical systems in direct application to signal processing (Patil and Sen, 2007)

Quantum mechanically, information is associated with sets of quantum probabilities which is the square of the quantum mechanical wave functions (Jaynes, 1957). Entropic measures and information theory in general provides a clear understanding for the stability of quantum systems. Shannon developed information theory in 1940's to find the fundamental limits on signal processing operation as well as storing communication data (Shannon, 1948). Shannon, Fisher and other quantum information entropies usually measures the spread of probability distribution for allowed quantum mechanical state in a D-dimensional space (Hohenberg and Kohn, 1964; Sears *et.al.*, 1980, Ikhdair and Sever, 2008). Entropic measurement under quantum information has a direct relationship with Heinsernberg uncertainty principle. The new Shannon entropy was first introduced by Beckner, Bialynicki-Birula and Mycieslki (BBM) in 1975 (Serrano et.al, 2016). A lot of researchers have studied Shannon information theory with respect to some considerable potentials using different quantum mechanical approach. Najafizade *et. al*. (2016), investigated nonrelativistic Shannon information entropy using Kratzer potential.Liu (2007) studied the relationship between densities of Shannon entropy and Fisher information for atoms and molecules. Serrano *et .al.* (2015), studied Shannon entropy for a solitonic profile mass Schrodinger equation with a squared hyperbolic cosecant potential. Sun *et.al*. (2015), studied Shannon information entropies for position dependent mass Schrodinger problem in hyperbolic potential well. Dehesa *et.al*. (2001), studied the spreading of the quantum –mechanical probability cloud for the ground state of Morse and modified Poschl-Teller potentials by means of global and local information theoretic measures. Analytical evaluation of Shannon and other information entropies involves complicated integrals; therefore, the entropic integrals is analytical calculated using orthogonal hypergeometric polynomials (Dehesa *et.al.*2001). In order to obtain the Shannon information theory for any quantum mechanical potential, one needs to first obtain the energy eigen value and the normalized wave function of the potential using any convenient method like: Exact quantization, Supersymmetric quantum mechanics approach, factorization method, Nikiforov-Uvarov method, proper quantization method, asymptotic iteration method, formula method and many others (Villalba and Roja, 2006, Isonguyo *et.al.*, 2013; Ikot *et.al.*, 2011). Some of the quantum mechanical potentials that has been investigated for bound state solutions of Schrodinger and Klein-Gordon equations are: Kratzer potential, Tietz-Wei potential, Hyperbolic, Rosen-Morse, Deng –Fan and many others (Champion*et.al.* 2018, Mustafa and Dong 2006;Ikhdair and Sever 2006, Okon *et.al.*, 2017, Okon *et.al.*,2018). In this work, we developed a novel potential model called the trigonometric Yukawa plus inversely quadratic potential to study Shannon entropy under Schrodinger equation. We obtain the eigen solution of this novel potential via parametric Nikiforov-Uvarov method. Most of the hypergeometric entropic integrals are obtained using Mathematica software because of its complexity. All mathematical algorithm was carried out using MATLAB and MATHEMATICA. The proposed potential is given as

$$V(r) = -\frac{v_0 e^{-\alpha r}\sin\alpha}{re^{-\alpha r}\sin\alpha} - \frac{v_1}{r^2} \tag{1}$$

PARAMETRIC NIKIFOROV-UVAROV (NU) METHOD

The Nikiforov-Uvarov (NU) method was presented by Nikiforov and Uvarov (Nikiforov and Uvarov 1988) to solve linear second-order generalized parametric of hypergeometric-type of the form

$$\psi''(s) + \frac{c_1 - c_2 s}{s(1-c_3 s)}\psi'(s) + \frac{1}{s^2(1-c_3 s)^2}\left[-\xi_1 s^2 + \xi_2 s - \xi_3\right]\psi(s) = 0 \tag{2}$$

With the following eigen solution total wave function

$$\psi(s) = N_n s^{c_{12}}(1-c_3 s)^{-c_{12}-\left(\frac{c_{13}}{c_3}\right)} P_n^{\left(c_{10}-1,\left(\frac{c_{11}}{c_3}\right)-c_{10}-1\right)}(1-2c_3 s) \tag{3}$$

and the eigen function equation is given as

$$c_2 n - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + n(n-1)c_3 + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \tag{4}$$

The parametric constants of equation (4) is expressed as

$$c_4 = \frac{1}{2}(1-c_1),\ c_5 = \frac{1}{2}(c_2 - 2c_3),\ c_6 = c_5^2 + \xi_1,\ c_7 = 2c_4 c_5 - \xi_2,\ c_8 = c_4^2 + \xi_3,\ c_9 = c_3 c_7 + c_3^2 c_8 + c_6,$$
$$c_{10} = c_1 + 2c_4 + 2\sqrt{c_8},\ c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right),\ c_{12} = c_4 + \sqrt{c_8},\ c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right) \tag{5}$$

Radial Solution of Schrodinger Equation

One dimensional Schrodinger wave equation is given as

$$\frac{d^2\Psi}{dr^2} + \frac{2\mu}{\hbar^2}\left[E_{nl} + \frac{v_0 e^{-\alpha r}\sin\alpha}{re^{-\alpha r}\sin\alpha} + \frac{v_1}{r^2} - \frac{\hbar^2 l(l+1)}{2\mu r^2}\right] = 0 \tag{6}$$

The elegant approximation to the centrifugal term is expressed as

$$\frac{1}{r^2} = \frac{4\alpha^2 e^{-2\alpha r}\sin^2\alpha}{\left(1-e^{-\alpha r}\sin\alpha\right)^2} \Rightarrow \frac{1}{r} = \frac{2\alpha e^{-\alpha r}\sin\alpha}{\left(1-e^{-\alpha r}\sin\alpha\right)} \tag{7}$$

Substituting equation (7) into (6) gives

$$\frac{d^2\Psi}{dr^2}+\frac{2\mu}{\hbar^2}\begin{bmatrix} E_{nl}+\dfrac{2\alpha v_0 e^{-2\alpha r}\sin^2\alpha}{(e^{-\alpha r}\sin\alpha)(1-e^{-\alpha r}\sin\alpha)}+\dfrac{4v_1\alpha^2 e^{-2\alpha r}\sin^2\alpha}{(1-e^{-\alpha r}\sin\alpha)^2} \\ -\dfrac{\hbar^2 l(l+1)}{2\mu}\dfrac{4\alpha^2 e^{-2\alpha r}\sin^2\alpha}{(1-e^{-\alpha r}\sin\alpha)^2} \end{bmatrix}=0 \tag{8}$$

By setting $s=e^{-\alpha r}\sin\alpha$ and with mathematical simplification, equation (8) can be transform into a differential in S-coordinate as

$$\frac{d^2\Psi}{ds^2}+\frac{(1-s)}{s(1-s)}\frac{d\Psi}{ds}+\frac{1}{s^2(1-s)^2}\left\{-\left(\varepsilon^2+\delta_0+4l(l+1)\right)s^2+\left(2\varepsilon^2+\delta_0\right)s-\left(\varepsilon^2-\chi_1\right)\right\} \tag{9}$$

Where

$$\varepsilon^2=-\frac{2\mu E_{nl}}{\hbar^2\alpha^2},\quad \delta_0=\frac{4\mu v_0}{\hbar^2\alpha}\quad,\quad \chi_1=\frac{8v_1\mu}{\hbar^2} \tag{10}$$

Comparing equation (9) to (2) and by using equation (5) the following parametric constants are obtained.

$$\begin{aligned} &c_1=c_2=c_3=1,\quad c_4=0,\quad c_5=-\frac{1}{2},\quad c_6=\frac{1}{4}+\varepsilon^2+\delta_0;\quad c_7=-2\varepsilon^2-\delta_0\sqrt{b^2-4ac}\,\frac{1}{2}\\ &c_8=\varepsilon^2-\chi_1,\quad c_9=\frac{1}{4}-\chi_1+4l\left(l+1\right),\quad c_{10}=1+\sqrt{\varepsilon^2-\chi_1}\\ &c_{11}=2+2\sqrt{1-4\chi_1+16l\left(l+1\right)}+2\sqrt{\varepsilon^2-\chi_1};\quad c_{12}=\sqrt{\varepsilon^2-\chi_1}\\ &c_{13}=-\frac{1}{2}-\frac{1}{2}\sqrt{1-4\chi_1+16l\left(l+1\right)}-\sqrt{\varepsilon^2-\chi_1} \end{aligned} \tag{11}$$

Using equation (4) the energy eigen equation is given as

$$E_{nl}=-\frac{\hbar^2\alpha^2}{2\mu}\left[\frac{\left(n^2+n+\frac{1}{2}\right)-\frac{4\mu v_0}{\hbar^2\alpha^2}-\frac{16\mu v_1}{\hbar^2}+\left(n+\frac{1}{2}\right)\sqrt{1-\frac{32\mu v_1}{\hbar^2}+4l(l+1)}}{\left(2n+1\right)+\sqrt{1-\frac{32\mu v_1}{\hbar^2}+4l(l+1)}}\right]^2-4v_1\alpha^2 \tag{12}$$

By the help of equation (3), the total wave function is given as

$$\Psi_n(s)=N_n s^{\sqrt{\varepsilon^2+\chi_1}}\left(1-s\right)^{-\frac{1}{2}-\left(\sqrt{\frac{1}{4}-\chi_1+4l(l+1)}+\sqrt{\varepsilon^2+\chi_1}\right)}P_n^{\left[\left(1+2\sqrt{\varepsilon^2-\chi_1}\right),\left(2+2\sqrt{1-4\chi_1+16l(l+1)}\right)+2\sqrt{\varepsilon^2-\chi_1}\right]}\left(1-2s\right) \tag{13}$$

Analytical Calculation of Shannon Entropy

Shannon entropy which gives the measure of the single particle density is presented in equation (14) and (15) for position and momentum spaces respectively as

$$s_x=-\int_{-\infty}^{+\infty}\rho(x)\log\rho(x)dx \tag{14}$$

$$s_p=-\int_{-\infty}^{+\infty}\rho(p)\log\rho(p)dp \tag{15}$$

Where $\rho(x)=\left|\Psi(x)\right|^2$ *and* $\rho(p)=\left|\Psi(p)\right|^2$. The Shannon entropy for position space as presented in equation (14) can further be expressed as

$$\begin{aligned} s_x &= -\int_{-\infty}^{+\infty}\left(\frac{1}{d^2}\left|P_n(x)\right|^2\omega(x)\right)\log\left(\frac{1}{d^2}\left|P_n(x)\right|^2\omega(x)\right)dx \\ \Rightarrow s_x &= \log\left(d_n^2\right)+\frac{1}{d^2}\left(E_n+I_n\right) \\ E_n &= -\int_{-\infty}^{+\infty}\left(\left|P_n(x)\right|^2\omega(x)\right)\log\left(\left|P_n(x)\right|^2\right)dx \\ I_n &= -\int_{-\infty}^{+\infty}\left(\left|P_n(x)\right|^2\omega(x)\right)\log\left(\omega(x)\right)dx \end{aligned} \tag{16}$$

Here, E_n and I_n are called the entropic integrals usually expressed in terms of Jacobi polynomial.

Evaluating Shannon entropy for position and momentum space analytically first involve getting the probability density which is the square of the wave function before finding the entropic integrals expressing in hypergeometric function of associated Jacobi polynomials. The total wave function as presented in equation (13) can be simplify for easy normalization. Starting from equation (13), let $\sigma_1=\sqrt{\varepsilon^2-\chi_1}$ and $\sigma_2=\sqrt{\frac{1}{4}-\chi_1+4l\left(l+1\right)}$, equation (13) now becomes

$$\Psi_n(s)=N_n s^{\sigma_1}(1-s)^{-\frac{1}{2}-(\sigma_1+\sigma_2)}P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s) \tag{17}$$

Squaring equation (17), the probability density is then obtained as

$$\rho(x)=\rho(r)=N_n^2 s^{2\sigma_1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2 \tag{18}$$

From the fact that $s=e^{-\alpha r}\sin\alpha$, then integration boundaries from $(-\infty,+\infty)$ in r-dimension changes to $(0,1)$ and by making use equation (14), the Shannon entropy for position space for the proposed potential is given as

$$S_x^{TYPIQP}=-\frac{N_n^2}{2\alpha}\int_0^1\left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times \\ \log N_n^2\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\end{array}\right]ds \tag{19}$$

Using logarithm expansion equation (19) can be reduced to

$$S_x^{TYPIQP}=-\frac{N_n^2}{2\alpha}\int_0^1\left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times \\ \left(\begin{array}{l}\left(\log N_n^2\right)+\log\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)+ \\ \log\left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\end{array}\right)\end{array}\right]ds \tag{20}$$

Equation (20) can further be separated into three separate integrals as shown below

$$S_x^{TYPIQP} = -\log\left(N_n^2\right) \times \frac{N_n^2}{2\alpha}\int_0^1 \left[\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\right]ds$$
$$-\frac{N_n^2}{2\alpha}\int_0^1 \left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\times \\ \left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times \\ \log\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\end{array}\right] - \frac{N_n^2}{2\alpha}\int_0^1 \left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\times \\ \left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times \\ \log\left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\end{array}\right] \quad (21)$$

The first integral of equation (30) signifies normalization with a unique value of one. That is

$$\frac{N_n^2}{2\alpha}\int_0^1 \left[\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\right]ds = 1 \quad (22)$$

Equation (22) finally reduced to

$$S_x^{TYPIQP} = -\log\left(N_n^2\right) + \frac{N_n^2}{2\alpha}\left[E\left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|\right) + I\left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|\right)\right] \quad (23)$$

Where E and I are entropic integrals with the following expressions

$$E\left[\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|\right] = -\int_0^1 \left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\times \\ \left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times \\ \log\left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\end{array}\right]ds \quad (24)$$

$$I\left[\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|\right] = -\int_0^1 \left[\begin{array}{l}\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\times \\ \left(\left|P_n^{\left[(1+2\sigma_1),2+2(\sigma_1+\sigma_2)\right]}(1-2s)\right|^2\right)\times\log\left(s^{2\sigma_1-1}(1-s)^{-1-2(\sigma_1+\sigma_2)}\right)\end{array}\right]ds \quad (25)$$

Guerrero and Aptekarev [17] define entropic integrals expressed in terms of digamma functions as follows:

$$E_n\left[P_n^{[\alpha,\beta]}(\alpha)\right] = -\int_a^b \omega(\alpha,\beta)\left|P_n^{[\alpha,\beta]}(\alpha)\right|^2 \log\left|P_n^{[\alpha,\beta]}(\alpha)\right|^2 dx = \log(\pi) - 1 - (\alpha+\beta)\log 2 + o(1) \quad (26)$$

$$I\left[P_n^{[\alpha,\beta]}(\alpha)\right] = -\int_{-1}^1 \omega(\alpha,\beta)\left|P_n^{[\alpha,\beta]}(\alpha)\right|^2 \log\omega(\alpha,\beta)\,dx =$$
$$-\alpha\psi(n+\alpha+1) - \beta\psi(n+\beta+1) + (\alpha+\beta)\times \quad (27)$$
$$\left[-\log 2 + \frac{1}{(2n+\alpha+\beta+1)} + 2\psi(2n+\alpha+\beta+1) - \psi(n+\alpha+\beta+1)\right]$$

In order to express E_n as a digamma function, we consider equation (23) such that

$$\alpha = 2\sigma_1 + 1; \quad \beta = 2 + 2(\sigma_1+\sigma_2); \quad x = 1-2s$$

By making use of equation (26), the entropic integral

$$E_n\left[P_n^{[\alpha,\beta]}(\alpha)\right] = \log\pi - (4\sigma_1 + 2\sigma_2 + 3)\log 2 - 1 + o(1) \quad (28)$$

$$I\left[P_n^{[\alpha,\beta]}(\alpha)\right]=-\left(1+2\sigma_1\right)\psi\left(n+2\sigma_1+2\right)-\left(2+2\sigma_1+2\sigma_2\right)\psi\left(n+2+2\sigma_1+2\sigma_2\right)$$
$$+\left(3+4\sigma_1+2\sigma_2\right)\left[\begin{array}{l}-\log 2+\dfrac{1}{\left(2n+4\sigma_1+2\sigma_2+4\right)}+2\psi\left(2n+4\sigma_1+2\sigma_2+3\right)\\ -\psi\left(n+4\sigma_1+2\sigma_2+4\right)\end{array}\right] \quad (29)$$

Substituting equation (28) and (29) into (23) gives Shannon entropy for position space as

$$S_x^{TYPIQP}=-\log\left(N_n^2\right)+\frac{N_n^2}{2\alpha}\left[\begin{array}{l}\log\pi-\left(4\sigma_1+2\sigma_2+3\right)\log 2\\ -1+o(1)-\left(1+2\sigma_1\right)\psi\left(n+2\sigma_1+2\right)\\ -\left(2+2\sigma_1+2\sigma_2\right)\psi\left(n+2+2\sigma_1+2\sigma_2\right)\\ +\left(3+4\sigma_1+2\sigma_2\right)\left[\begin{array}{l}-\log 2+\dfrac{1}{\left(2n+4\sigma_1+2\sigma_2+4\right)}\\ +2\psi\left(2n+4\sigma_1+2\sigma_2+3\right)\\ -\psi\left(n+4\sigma_1+2\sigma_2+4\right)\end{array}\right]\end{array}\right] \quad (30)$$

To obtain the normalized function for position space Shannon entropy, the normalization constant can be evaluated using the indefinite integrals

$$\int_{-\infty}^{+\infty}\Psi(x)\Psi^*(x)dx=\int_{-\infty}^{+\infty}\left|\Psi(x)\right|^2dx=1 \quad (31)$$

Substituting (17) into (31) and using Mathematica, the normalization constant for $n=0$ is given as

$$N_0^{TYPIQP}=\frac{1.41421}{\left[\dfrac{\Gamma\left(2\sigma_1\right)\left(-\sin\alpha\right)^{-2\sigma_1}\left(-\sin^2\alpha\right)^{-2\sigma_1}{}_2F_1\left(2\sigma_1,2\sigma_1+2\sigma_2+1;2\sigma_1+1;\sin\alpha\right)}{\alpha\Gamma\left(2\sigma_1+1\right)}\right]^{0.5}} \quad (32)$$

Lets note that the Jacobi polynomial for $n=0$ is unity, therefore the total normalized ground state wave function is given as

$$\Psi_0^{TYPIQP}(r)=\left\{\frac{1.41421}{\left[\dfrac{\Gamma\left(2\sigma_1\right)\left(-\sin\alpha\right)^{-2\sigma_1}\left(-\sin^2\alpha\right)^{-2\sigma_1}\times{}_2F_1\left(2\sigma_1,2\sigma_1+2\sigma_2+1;2\sigma_1+1;\sin\alpha\right)}{\alpha\Gamma\left(2\sigma_1+1\right)}\right]^{0.5}}\right\}\times s^{\sigma_1}(1-s)^{-\frac{1}{2}-\left(\sigma_1+\sigma_2\right)} \quad (33)$$

The corresponding normalized ground state wave function in momentum space is obtained by taking the fourier transform of the position space. The fourier expression is given as

$$\Psi_0^{TYPIQP}(p)=\frac{1}{\sqrt{2\pi}}\int_0^{\infty}\Psi_0^{TYPIQP}(r)e^{-ipr}dr \quad (34).$$

The corresponding normalized momentum space wave function is obtained using mathematica software as

$$\Psi_0^{TYPIQP}(p)=\left\{\frac{\left[\begin{array}{c}0.282095(-\sin\alpha)^{-\sigma_1}\left(-\sin^2\alpha\right)^{-\sigma_1}\Gamma\left(\frac{ip}{2\alpha}+\sigma_1\right)\times\\ {}_2F_1\left(\frac{ip}{2\alpha}+\sigma_1,\sigma_1+\sigma_2+0.5;\frac{ip}{2\alpha}+\sigma_1+1;\sin\alpha\right)\end{array}\right]}{\alpha\Gamma\left(\frac{ip}{2\alpha}+\sigma_1+1\right)\left[\frac{\begin{array}{c}\Gamma(2\sigma_1)(-\sin\alpha)^{-2\sigma_1}\left(-\sin^2\alpha\right)^{-2\sigma_1}\times\\ {}_2F_1(2\sigma_1,2\sigma_1+2\sigma_2+1;2\sigma_1+1;\sin\alpha)\end{array}}{\alpha\Gamma(2\sigma_1+1)}\right]^{0.5}}\right\} \quad (35)$$

Numerical Computation of the Energy Eigen Equation.

We implemented MATLAB algorithm using equation (12) to compute the numerical bound state solution of the proposed potential. We adopted the following parameters $v_0=v_1=0.01;\quad \mu=\hbar=1;\quad \alpha=0.01,0.02,0.03,0.04\ \ and\ 0.05$ as presented in tables 1 and 2.

Table 1: Numerical energy eigen solution for $\alpha=0.01,0.02\ \ and\ 0.03$

n	l	$E_{nl}(eV)$ $\alpha=0.01$	n	l	$E_{nl}(eV)$ $\alpha=0.02$	n	l	$E_{nl}(eV)$ $\alpha=0.03$
0	0	93.8874875	0	0	23.5749500	0	0	10.5540542
1	0	-43.2847369	1	0	-10.7593556	1	0	-4.73632791
2	0	-24.5639331	2	0	-6.07372535	2	0	-2.65002274
3	0	-14.1881063	3	0	-3.47693682	3	0	-1.49412834
4	0	-9.01078806	4	0	-2.18142888	4	0	-0.917917691
5	0	-6.16449169	5	0	-1.46946953	5	0	-0.601760981
6	0	-4.45516192	6	0	-1.04218669	6	0	-0.412560606
7	0	-3.35503349	7	0	-0.767480708	7	0	-0.291488215
8	0	-2.60774072	8	0	-0.581183245	8	0	-0.209967528
0	1	58.7382293	0	1	14.7330194	0	1	6.58374657
1	1	2.70253815	1	1	0.749215104	1	1	0.387195211
2	1	-9.45016876	2	1	-2.28368125	2	1	-0.957074422
3	1	-8.75170775	3	1	-2.10966154	3	1	-0.880506526
4	1	-6.70599362	4	1	-1.59952849	4	1	-0.655183963
5	1	-5.04679164	5	1	-1.18594333	5	1	-0.472823476
6	1	-3.85426758	6	1	-0.888911434	6	1	-0.342289656
7	1	-3.00522230	7	1	-0.677688456	7	1	-0.249959357
8	1	-2.39093904	8	1	-0.525144118	8	1	-0.183810869
0	2	27.2782420	0	2	6.84125519	0	2	3.05642712
1	2	11.1764005	1	2	2.84707908	1	2	1.30417891
2	2	0.465093158	2	2	0.189938062	2	2	0.138247276
3	2	-3.11730193	3	2	-0.698961000	3	2	-0.252216537
4	2	-3.70530983	4	2	-0.845192303	4	2	-0.317086231
5	2	-3.39149001	5	2	-0.767817890	5	2	-0.284051196
6	2	-2.89101200	6	2	-0.644236477	6	2	-0.230928343
7	2	-2.41462873	7	2	-0.526725589	7	2	-0.180649495
8	2	-2.01162574	8	2	-0.427510574	8	2	-0.138577364

Table 2: Numerical energy eigen solution for $\alpha = 0.04, and \quad 0.05$

n	l	$E_{nl}(eV)$ $\alpha = 0.04$	n	l	$E_{nl}(eV)$ $\alpha = 0.05$
0	0	5.99667500	0	0	3.88718750
1	0	-2.62849388	1	0	-1.65311543
2	0	-1.45221107	2	0	-.898326665
3	0	-.801021191	3	0	-.481172485
4	0	-.477088499	4	0	-.274583256
5	0	-.300118367	5	0	-.162755972
6	0	-0.195033994	6	0	-0.0974688974
7	0	-0.128651880	7	0	-0.574083143
8	0	-0.0848529073	8	0	-0.0322209350
0	1	3.73131224	0	1	2.41083546
1	1	0.260141667	1	1	0.200953776
2	1	-0.493376860	2	1	-0.279425684
3	1	-0.451310519	3	1	-0.253760272
4	1	-0.326195660	4	1	-0.175602205
5	1	-0.225419274	5	1	-0.113306742
6	1	-0.153946716	6	1	-0.0700344911
7	1	-0.104147203	7	1	-0.0409282157
8	1	-0.0692869713	8	1	-0.0217016590
0	2	1.73150000	0	2	1.11798767
1	2	0.763652066	1	2	0.512903861
2	2	0.119285938	2	2	0.109555511
3	2	-0.0971511544	3	2	-0.0267990324
4	2	-0.134074515	4	2	-0.0513689714
5	2	-0.117211720	5	2	-0.0427085487
6	2	-0.0895324106	6	2	-0.0276650787
7	2	-0.0636992860	7	2	-0.0141501810
8	2	-0.0426735783	8	2	-0.00401402542

RESULTS AND DISCUSSION

The numerical solutions were carried out for a various principal quantum numbers $n = 0$ to $n = 8$, but with fixed orbital angular quantum number $l = 0, 1 and \quad 2$. The numerical solutions is predominantly negative which is one of the necessary and sufficient condition for bound state solution. These results authenticate the theoretical frame work as reported in an existing literature. Some of the numerical solutions are negative which signify that the potential model is suitable for describing both particles and antiparticles. The solution also increases with an increase in quantum state for various orbital angular quantum number; which is a significant trend for complete description of bound state solutions.

CONCLUSION

In this work, we developed a potential model called trigonometric Yukawa plus inversely quadratic potential to study bound study bound state solution of Schrodinger equation using Nikiforov-Uvarov method. We extended the work to compute analytically the Shannon entropy for position and momentum space using the proposed potential. The momentum space entropy was obtained by taking the Fourier transform of the position space. The normalization constant was obtained using Mathematica software. The numerical bound state solution increase with an increase in quantum state for various orbital angular quantum number.

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