

# INTERVAL TYPE-2 FUZZY FRAMEWORK FOR URBAN TRIP DISTRIBUTION MODELING



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## ABSTRACT

Trip distribution modeling (TDM) is one of the most active parts of travel demand analysis in transportation planning (TP). Due to the nature of transport data and the weakness of existing gravity model, most transport decisions take place under subjectivity, ambiguity, uncertainty and imprecision, and are difficult to be measured by crisp values. An Interval Type-2 Fuzzy Logic (IT2FL) IT2FL framework is designed and applied to model inter-city passenger flow in urban TDM to estimate the volume of trip interaction between selected zones. Six (6) urban centers in Akwa Ibom State, Nigeria, were selected for the study. The data on passengers' trips generated, passengers trips attracted and the distance used in the study were obtained from travel survey. Four performance metrics namely, mean absolute error (MAE), mean square error (MSE), root mean square error (RMSE) and mean absolute percentage error (MAPE) were applied and results indicate that IT2FL model gives a better performance with lower error values of 10.2, 1.18, 0.14 and 0.22 for MAE, MSE, RMSE and MAPE, respectively compared to 11.66, 2.25, 0.18 and 0.32, respectively for the gravity model. Thus, IT2FLS can be applicable in transportation problem to aid good decision making for both travelers and transport planners.

## INTRODUCTION

Trip distribution modeling (TDM), which constitutes the second stage in the traditional transportation planning, is one of the most active parts of travel demand analysis in transportation planning (TP) process used to determine the number of trips between pairs of zones when the number of trips generated attracted by particular zones is known.. Transportation is an essential part in social, industrial and economical process which helps shape an economic health and provide for the mobility of people and goods. Effective transportation systems lead to the efficient movement of goods and people, which significantly contribute to the quality of life in every society. The prediction of trip distribution involves the prediction of flows in a network regardless of a possible transportation mode or travel route (Kalic and Teodorovic, 2003).

Transportation planning process is a wide human-oriented field with diverse and challenging problems waiting to be solved (Black, 2003). In the real world, most of these issues are characterized by uncertainty, ambiguity or imprecise sets of information, which can be difficult to translate into numerical values, called crisp values whereby simplified mathematical models are not adequate for their analysis. Techniques based on intelligent computing such as Fuzzy logic (FL) is useful in trip distribution modeling. The use of fuzzy set theory in real-world problems is very promising in modeling spatial interactions in transport planning because of its ability to tolerate imprecision, uncertainty and lack of information to achieve robustness in decision making with low cost (Jassbi, 2011). In this work, an Interval Type-2 Fuzzy Logic (IT2FL) IT2FL framework is designed and applied to model inter-city passenger flow in urban TDM to estimate the volume of trip interaction between selected zones. IT2FLS as applied in the study gives satisfactory and better results when compared with the existing gravity model and can be applicable in current and future scenarios.

Transportation planning process takes community needs and expectations into consideration and establishes a way of designing future transportation systems (Boyce, 2002). One of the

most important stages of transportation planning process is forecasting future travel demand in a desired level of accuracy. A number of deterministic and stochastic models have been developed to understand travel behaviour better and to achieve more accurate forecasts. The traditional urban travel demand modeling consists of a sequential procedure often referred to as the ‘four-step’ modeling process: *trip generation, trip distribution, modal choice and trip assignment* (Casey (1955) (Taaffe et al., 1996) (Meyer and Miller, 2001) (Teodorovic and Kalic, 1996) (Kalic and Teodorovic, 1997) (Jassbi, 2011).

The IT2FLS is an extension of type-1 fuzzy logic (T1FL) developed by (Zadeh, 1964) to adequately deal with the uncertainties about the fuzzy membership value itself (Zadeh in 1975). The uncertainty in data can be represented by the footprint of uncertainty (FOU with ability, reliability, capability and robustness and has been applied to different fields of human existence. An IT2F set,  $\tilde{A}$  is characterized by a membership interval in the universe of discourse  $X$  as;

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (1)$$

$$\tilde{A} = \sum_{i=1}^p [\sum_{u \in J_x} [1/u]] / x_i \quad (2)$$

Where  $x$ , the *primary variable*, has domain  $X$ ;  $u \in U$ , the *secondary variable*, has domain  $J_x$  at each  $x \in X$ ;  $J_x$  is called the primary membership of  $x$  and the secondary grades of  $\tilde{A}$  all equal 1 (Mendel, 2001) (Mendel et al, 2002). Uncertainty about  $\tilde{A}$  is conveyed by the union of all the primary memberships, which is called the *footprint of uncertainty* (FOU) of  $\tilde{A}$ , encompassing all the embedded primary membership functions  $J_x$  of  $\tilde{A}$  as shown in 3.

$$\mu_{\tilde{A}}(x, u) = 1, FOU(\tilde{A}) = \cup_{x \in X} J_x = \{(x, u) : u \in J_x \subseteq [0, 1]\} \quad (3)$$

FOU( $\tilde{A}$ ) is bounded by *upper membership function* (UMF)  $\bar{\mu}_{\tilde{A}}(x)$  and *lower membership function* (LMF)  $\underline{\mu}_{\tilde{A}}(x)$ ,  $\forall x \in X$ , respectively assuming minimum and maximum of the membership functions of the embedded T1FSs in the FOU.  $\bar{\mu}_{\tilde{A}}(x) \equiv \overline{FOU(\tilde{A})} \forall x \in X$  and  $\underline{\mu}_{\tilde{A}}(x) \equiv \underline{FOU(\tilde{A})} \forall x \in X$ . For IT2FS,  $J_x = [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)]$ ,  $\forall x \in X$ . Fuzzification maps inputs to fuzzy values using membership function (MF). The study uses IT2 Gaussian membership function (GMF) because of its suitability for a highly dynamic random system, with fixed mean, and uncertain standard deviation,  $\sigma$  to calculate the degree of membership of the input variables.

$$\mu_A(x) = \exp\left(-\frac{x-w}{2\sigma}\right) \quad \sigma \in [\sigma_1, \sigma_2] \quad (4)$$

The upper and lower membership functions are calculated using:

$$\bar{\mu}_{\tilde{A}_{im}}(x_i) = \exp\left(-\frac{x_i-w_{im}}{2\sigma_{2,im}^2}\right), \bar{\mu}_{\tilde{A}}(x) = N(w, \sigma_2; x) \quad (5)$$

$$\underline{\mu}_{\tilde{A}_{im}}(x_i) = \exp\left(-\frac{x_i-w_{im}}{2\sigma_{1,im}^2}\right), \underline{\mu}_{\tilde{A}}(x) = N(w, \sigma_1; x) \quad (6)$$

Where  $w$  is the center (mean) of the MF,  $\sigma$  is the standard deviation of the MF and  $x$  is the input vector. The variables  $\bar{\sigma}_{2,im}$  and  $\underline{\sigma}_{1,im}$  are premise parameters that define the degree of membership of each element to the fuzzy set  $\tilde{A}$  and FOUs of the IT2FS. The detail description is found in (Liang and Karnik, 2000) (Mendel, 2001). For an IT2FLS with  $m$  inputs and *one* output have the if-then rules specified as in equation (7).

$$IF x_i \text{ is } \tilde{D}_i^l \text{ AND } \dots \text{ AND } x_m \text{ is } \tilde{D}_m^l \text{ THEN } y \text{ is } \tilde{E}^l \quad (7)$$

Where  $x_i, i = 1, \dots, m$  are the antecedents,  $y_j, j = 1, \dots, n$  are the consequents of the  $l$ th rule,  $l = 1, \dots, P$  of IT2FLS. The  $\tilde{D}^l$ 's are the MFs  $\mu_{\tilde{D}_i^l}(x_i)$  of the antecedent part assigned to the  $i$ th input  $x_i$ , The  $\tilde{E}^l$ 's are the MFs  $\mu_{\tilde{E}_j^l}(y_j)$  of the consequent part assigned to the  $j$ th output  $y_j$ . The

inference engine combines the fired rules to give a mapping from input IT2FSs to output IT2FSs. The firing intervals for lower and upper membership functions are evaluated using (Hagras, 2007) as;  $F^i(x^i) = [\underline{\mu}_{\bar{f}_1^i}(x_1^i) * \dots * \underline{\mu}_{\bar{f}_m^i}(x_1^i)], [\bar{\mu}_{\bar{f}_1^i}(x_1^i) * \dots * \bar{\mu}_{\bar{f}_m^i}(x_1^i)] \equiv [f^i, \bar{f}^i], i = 1, 2, \dots, M]$  (8)

Where  $F^i(x^i)$  is the antecedent of rule  $i$  and  $\mu_{F^i}(x^i)$  is the degree of membership of  $x$  in  $F^i(x)$ .  $\bar{\mu}_{\bar{f}_1^i}(x)$  and  $\underline{\mu}_{\bar{f}_m^i}(x)$  are upper and lower MFs of  $\mu_{f^i}$ . In IT2FLS, an exact iterative method of type reduction is performed by combining  $F^i(x^i)$  and its consequent to compute the centroid of an IT2FS. to produce type-reduced sets (T1FS) using (9) (10) and (11) respectively.

$$Y_{TR}(x^i) = [y_l(x^i), y_r(x^i)] \equiv [y_l, y_r] = \bigcup_{y^i \in Y^i} \frac{\sum_{i=1}^N f^i y^i}{\sum_{i=1}^N f^i} \quad (9)$$

$$y_l = \min_{L \in [1, N-1]} \frac{\sum_{n=1}^L \bar{f}^n y^n + \sum_{n=L+1}^N \underline{f}^n y^n}{\sum_{n=1}^L \bar{f}^n + \sum_{n=L+1}^N \underline{f}^n} = \frac{\sum_{i=1}^N f_l^i y_l^i}{\sum_{i=1}^N f_l^i} \quad (10)$$

$$y_r = \max_{R \in [1, N-1]} \frac{\sum_{n=1}^R \underline{f}^n y^n + \sum_{n=R+1}^N \bar{f}^n y^n}{\sum_{n=1}^R \underline{f}^n + \sum_{n=R+1}^N \bar{f}^n} = \frac{\sum_{i=1}^N f_r^i y_r^i}{\sum_{i=1}^N f_r^i} \quad (11)$$

Where  $y_l^i$  and  $y_r^i$  are the left and right end points of the centroid of the consequent of the  $i$ th rule while  $f_l^i$  and  $f_r^i$  are the lower and upper firing degrees of the  $i$ th rule and  $N$  is the number of fired rules. Defuzzification of the interval set is performed using the average of  $y_{lk}$  and  $y_{rk}$ , and the defuzzified crisp output for each output  $k$  is achieved;

$$Y(X) = \frac{y_l + y_r}{2} \quad (12)$$

### METHODOLOGY

A survey was carried out using one thousand eight hundred and fifty-six (1,856) copies of questionnaire coded and the analyzed data is presented in Table 2. The structure of interval type-2 fuzzy intelligent framework for urban trip distribution modeling is shown in Figure 1. In this stage, firstly, fuzzification is carried out based on equations (4) (5) and (6). The universe of discourse is presented in Table 1. Fuzzy sets of the inputs and output variables and their associated values and labels are defined as shown Table 2 respectively. 74 fuzzy rules are defined by the experts using (7). Inference Engine mechanism is performed and type-reduction is computed using (8) (9) and (10) respectively. Finally, defuzzification is computed using (11) to obtain crisp value (Trip Interaction) in (12) respectively. The study employs Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE) for the purpose of performance measure and model evaluation.

Table1: Domain Intervals of Input and Output Variables

Variables	Lower Bound	Upper Bound
Trip production	0	350
Trip attraction	0	300
Distance	0	100
Trip Interaction	0	100

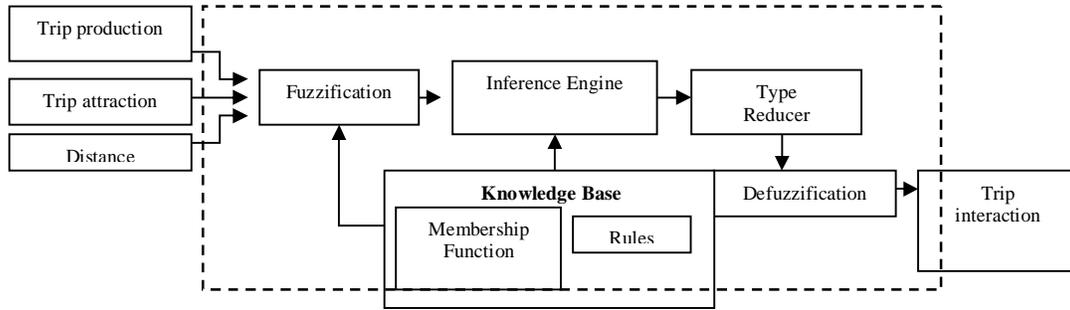


Figure 1: Intelligent Framework for Urban Trip Distribution Modeling

Table 2: IT2FL Input's/Output's Universe of Discourse for Urban trip Distribution

Variable		Mean Center (c)	Standard Deviation (σ)	
			Upper	Lower
<b>Production</b> <b>0-350</b>	Very Low	39.06	12.4	7.33
	Low	94.64	14.1	9.47
	Medium	151.4	13.8	8.02
	High	214.8	15.9	10.14
	Very High	276.0	14.07	9.17
<b>Attraction</b> <b>0-300</b>	Very Low	29.72	8.97	4.26
	Low	74.47	12.7	7.55
	Medium	122	10.7	5.89
	High	166	12.15	7.13
	Very High	216.9	12.2	7.58
<b>Distance</b> <b>0-100%</b>	Short	14.95	4.648	2.76
	Medium	39.17	6.53	4.51
	Long	68.8	10.72	7.77
<b>Interaction(Output)</b> <b>0-100%</b>	Very Low	0	10.62	7.4
	Low	25	10.62	7.61
	Medium	50	10.62	7.18
	High	75	10.62	7.66
	Very High	100	10.62	7.31

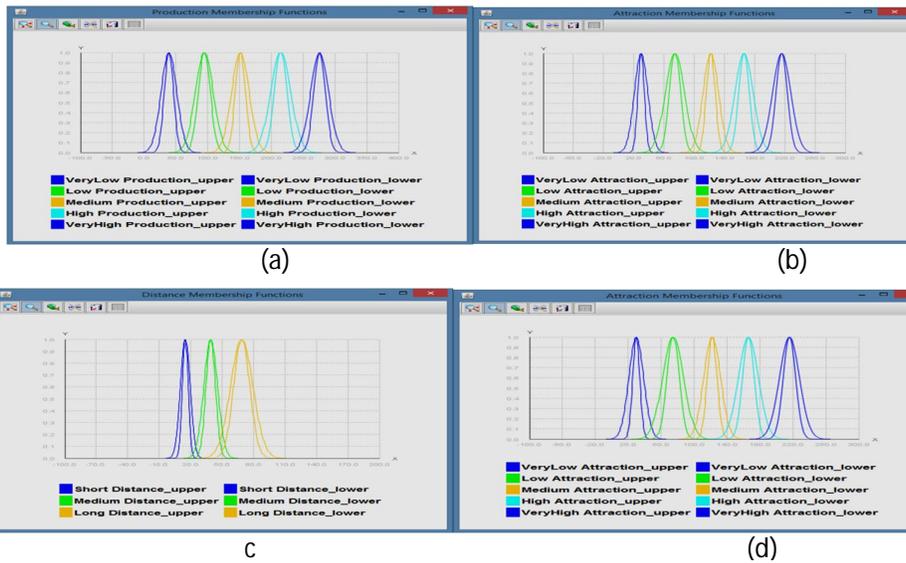
### RESULTS AND DISCUSSION

In this work, an IT2F intelligent framework is designed and applied to model inter-city passenger flow in six urban centers in Akwa Ibom State, Nigeria. The data on passengers trips generated, passengers trips attracted and the distance used in the study are obtained from travel survey. The input data are generated based on the six variables; *trip production*, *trip attraction* and *trip distance* while *trip interaction* is the desired output. The result of trip generation/production as analyzed and observed from the survey is shown in Table 1. The result of the analysis of the data collected for trip generation with their volume and the trip attraction including the distance in all the six selected zones is presented in Table 2. Fuzzy logic toolbox in Matlab 7.5.0 is used for the input and output membership functions plots as presented in Figure 2(a)-(d) respectively. Figure 3 shows the inputs and output interface result of a case where the total trip production of 305 and total attraction of 177.6 with distance 20.1km are selected, the result gives 74.2% strong interaction possibility and interaction surface plot is displayed in Figure 4. The results IT2FL and gravity model for urban trip distribution applied in the six zones is presented in Table 3. Gravity model is also applied in the work for comparison purpose. The results shows that Uyo to Ikot Ekpene has the highest trip interactions volume of 83%, followed with Uyo to Eket with 79% while Uyo to Abak gives 74% volume of interaction, etc. From Figure 4, the output surface indicates a very strong trip interaction between Uyo and Abak giving the interaction volume of 74%. Also, From Table 6, it is shown that the volume of interaction between Uyo and Ikot Ekpene ranks first. When the total trip

production is 250, total attraction 228.2 and distance 28.2km are selected, a very strong trip interaction is achieved with of 82.7 (83%) possibility.

Table 3: Trips purpose and volume of trip

S/N	Types of Trips	Volume of Trips	Percentage total
1	School Trip	373	20%
2	Work Trip	254	13.70%
3	Visit to Friends	248	13.40%
4	Business Trips	234	12.60%
5	Recreation Trips	179	9.60%
6	Medical Trips	146	8.00%
7	Religious Trips	117	6.30%
8	Burial Trips	104	5.60%
9	Market Trips	97	5.20%
10	Wedding Trips	89	4.80%
11	Other Trips	15	0.80%
	Total	1856	100.00%



Figures 2: Membership plots for the inputs and output variables (a) Total trip production (b) Total Trip Attraction (c) Distance between pair of zones and (d) Total trip attraction



Figure 3: Input and output interface between Uyo and Abak

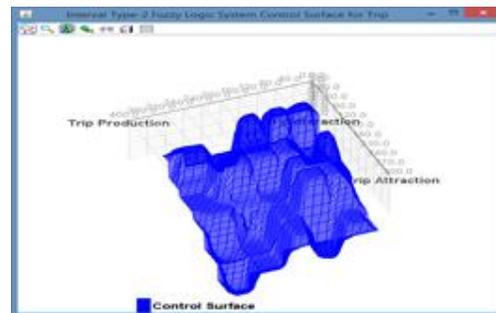


Figure 4: Output surface control between Uyo and Abak

From Table 3 it is observed that the IT2FL in urban trip distribution modeling demonstrates certain superiority over gravity model. The four performance measures applied in the study show considerably a better performance of the system gives MAE, MSE, RMSE and MAPE of 10.2, 1.18, 0.14 and 0.22 with IT2FL and 11.66, 2.25, 0.18 and 0.32 respectively.

Table 3: The results IT2FL and gravity model for urban trip distribution applied in the six zones

S/N	Origin/ Destination Of Trips	Trip Interaction Using Gravity Model	Trip Interaction Using Interval Type-2 Fuzzy Logic
1	Uyo-Abak	56	74
2	Uyo-Ikot Ekpene	83.	83
3	Uyo-Oron	77	68
4	Uyo-Ikot Abasi	68	75
5	Uyo-Eket	63	79
6	Abak-Ikot Ekpene	23	34
7	Abak-Oron	21	52
8	Abak-IkotAbasi	29	25
9	Abak-Eket	50	73
10	Ikot Ekpene-Oron	26	48
11	Ikot Ekpene-IkotAbasi	55	75
12	Ikot Ekpene-Eket	58	45
13	Oron-Eket	59	43
14	Oron- Ikot Abasi	63	74
15	IkotAbasi-Eket	27	50

### CONCLUSION

Trip distribution modeling (TDM) is one of the most active parts of travel demand analysis in transportation planning (TP). Most of the transport decisions take place under subjectivity, ambiguity, uncertainty and imprecision and partial truth. In this study, IT2FL is applied to model urban trip distribution problem in transportation planning in order to predict the trip interchange among the various urban centres in Akwa Ibom State. Results indicate that fuzzy logic could be used successfully to model situations in which people make decisions in an environment that is so complex that it is very hard to develop a mathematical model. In the future, the work can be implemented with more input variables and more zones. Also, the performance of the proposed system can be improved by integrating IT2FL with Ant colony optimization tool.

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