PROBABILISTIC INVENTORY CONTROL MODELS WITH VARIABLE DEMAND OVER A PLANNING HORIZON FOR CONSUMABLE PRODUCTS

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ABSTRACT

Probability distributions were used to model the movement of the demand of selected consumable products at a wholesale shop in Uyo, Nigeria. The demand rate of these products namely Biscuit, Noodles, Lacasera drink and Okpop (sweet and gum) for twelve months were obtained. The distribution for the demand for Biscuit, Noodles and Lacasera drink were shown to be normally distributed while Okpop followed Weibull distribution. Based on the distributions of demand, appropriate probabilistic inventory models for the items were obtained and the optimal order quantity (Q^*), probability of shortage as well as no shortage were obtained.

INTRODUCTION

Inventory management is pivotal for effective and efficient management of an organization. It is vital in the control of materials and goods that are held or stored for later use in the case of production or later exchange activities in the case of services. It is a critical management issue for most companies be it large, medium-sized, or small companies. The principal goal of inventory management involves balancing the conflicting economics of not wanting to hold too much stock and tie up capital and also to guide against incurring costs such as shortage, spoilage, pilferage and obsolescence but make items or goods available when and where required (quality and quantity wise) so as to avert the cost of not meeting such requirement, Toomey (2000). Essentially, inventory management involves planning and control. The planning aspect involves looking ahead in terms of the determination in advance what quantity of items to order, how often (periodicity) do we order for them to maintain the overall source – store sink and overall stock coordination in an economically efficient way. The control aspect, which is often described as stock control involves the application of the procedure setup at the planning stage to achieve the above objective. This may include monitoring stock levels periodically or continuously and deciding what to do on the basis of information that is gathered and adequately processed. Hence, effective inventory management can make a significant contribution to the largest item appearing on the asset side of the balance sheet, Hycinth *et al.* (2014). Therefore, effort must be made by the management of any organization to strike an optimum investment in inventory since it costs much money to tie down capital in excess inventory.

The development of modern inventory management principles began when Kotler (2000) derived the Economic Order Quantity (EOQ) formula. EOQ assumes that demand occurs at known, constant rate and supply fulfills the replenishment order after a fixed lead time. Unfortunately, the real world is not as ideal as that. In reality, demand rate is rarely constant, hard-to-predict market is common in most practical situations. Also, an unpredictable event in the supply system can cause and create unpredictable delays in replenishment. Moreover, in current times when outsourcing is at the center stage, complex and longer supply chains magnify the length and variability of lead times, Schroeder (2000). Morris (1995) and Drury (1996) stated that inventory management in its broadest perspective is to keep the most economical amount of one kind of asset in order to facilitate an increase in the total value of all assets of organization-human and material resources. Changes in inventory levels affect return on assets (ROA), which is an important financial parameter from an internal and external perspective. Reducing inventory usually improves ROA, and vice versa if inventory goes up without offsetting increases in revenue, Coyle, Bardi and Langley (2003). Inventory that corresponds to independent demand is called distribution inventory/finished product inventory while dependent demand inventory is known as manufacturing inventory/raw material inventory and work-in-process (WIP) inventory (Simchi-Levi*et al.*, 2004). Different approaches to managing inventory should be applied to align inventory supply with demand. Just-in-Time (JIT) approach and Material Requirement Planning (MRP) system are typically associated with managing manufacturing inventory to serve dependent demand. Cross-docking is a typical approach for managing distribution inventory efficiently while Vendor-managed-inventory (VIM) approach is applicable both for manufacturing inventory and distribution inventory. Probabilistic inventory model is used to plan inventory for items whose demand fluctuates and can be described using probability distribution. Fuerst (1981), Morris (1995),Nahmias (1997), Ray and Chaudhuri (1997),Minner (2000), Baker and Schroeder (2000), Hwang and Hann (2000), Goldsbyand Martichenko (2005) have contributed to the development and application of deterministic and probabilistic inventory models to solve managerial problems.

METHODOLOGY

Assumption of the Model

Assumption of the model are; the demand varies with time, the lead time is deterministic and there is continuous but not uniform depletion of stock.

Probability Distribution for the Demand of Products

The probability distribution of the demand of the products were not known and were assumed to be normally distributed with the density function:

$$f_D\left(D, \mu, \sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(D-\mu)^2}{2\sigma^2}}\ ;\ D \geq 0,\ \sigma > 0 \qquad (1)$$

Where D is the demand (random variable), μ and σ^2 are mean and variance of the normal distribution. This assumption was tested using the Shapiro-Wilk's Test using sample of quantity of product demanded for a 12 month period. Next, the chi-square goodness-of-fit test was used to ascertain the distribution of product(s) among the selected items that was/were shown not to be normally distributed.

Probability Inventory Model with Normally Distributed Demand

Let C_0 represents the unit cost of excess inventory and C_1 represents the unit cost of shortages as a result of unsatisfied demand.
Let D = demand (which is random and assumed to be normally distributed). That is $D \approx N(\mu, \sigma^2)$. Hence, the density function of D is given in (1).

The objective of this model is to choose the ordering quantity that will minimize total expected cost of excess inventory and cost of shortage as a result of unsatisfied demand. If Q is the quantity ordered and D is the quantity demanded, excess inventory occur when quantity ordered is greater than the quantity demanded; $Q > D$. Also, shortage in inventory occur when quantity demanded exceeds quantity ordered; $D > Q$.The cost function with continuous demand is given by;

$$C(Q, D) = \begin{cases} C_0(Q - D)\ if\ \ D > Q \\ C_1(D - Q)\ if\ \ D < Q \end{cases}$$

If Q is ordered; $C(Q) =$ cost of excess inventory + cost of shortage inventory

$$C(Q) = E[C(Q,D)] = \int_0^\infty C(Q,D) f(D) dD$$

$$= \int_0^Q C_0 (Q-D) f(D) dD + \int_Q^\infty C_1 (D-Q) f(D) dD \qquad (2)$$

Where $f(D)$ is the probability distribution function of demand. Equation (2) is the cumulative distribution of quantity to be ordered denoted by $F(Q)$. Hence,

$$F(Q) = \int_0^Q C_0 (Q-D) f(D) dD + \int_Q^\infty C_1 (D-Q) f(D) dD.$$

Let $G_1 = \int_0^Q C_0 (Q-D) f(D) dD$

and $G_2 = \int_Q^\infty C_1 (D-Q) f(D) dD$

$\therefore F(Q) = G_1 + G_2$

From (2), let $G(Q) = \int_0^Q C_0 (Q-D) f(D) dD - \int_Q^\infty C_1 (Q-D) f(D) dD$

By applying Leibniz rule,

$$\frac{d\int_{f_1(y)}^{f_2(y)} h(x,y) dx}{dy} = \int_{f_1(y)}^{f_2(y)} \frac{\partial h(x,y)}{\partial y} dx + h(f_2(y), y) \times f_2{}'(y) - h(f_1(y), y) f_1{}'(y)$$

Let $y = Q,\ f_2(y) = Q$ *and* $f_1(y) = 0$

$h(D,Q) = C_0 (Q-D) f(D)$

That is;

$$\frac{\partial h(D,y)}{\partial y} = \frac{\partial h(D,Q) f(D)}{\partial Q} = f(D)$$

$$h(f_2(y), y) = h(Q,Q) = (Q-Q) f(D) = 0;\ f_2{}'(y) = 1$$

$$h(f_1(y), y) = h(0,Q) = (Q-0) f(D) = Qf(D);\ f_1{}'(y) = 0$$

$$\frac{dG_1(Q)}{dQ} = C_0 \int_0^Q f(D) dD = C_0 p(D \le Q). \qquad (3)$$

Similarly,

$$\frac{dG_2(Q)}{dQ} = C_1 \int_Q^\infty f(D) dD = C_1 (1 - F(Q)) = C_1 p(D \ge Q) \qquad (4)$$

$$\frac{dG(Q)}{dQ} = 0 \Rightarrow C_0 \int_0^Q f(D) dD - C_1 \int_Q^\infty f(D) dD = 0 \Rightarrow C_0 F(Q) - C_1 (1 - F(Q)) = 0$$

$$(C_0 + C_1) F(Q) - C_1 = 0$$

$$F(Q) = \frac{C_1}{C_1 + C_0} \qquad (5)$$

$$F(Q) = p(D \le Q) = p(D - \mu \le Q - \mu) = p\left\{\frac{D-\mu}{\sigma} \le Z \le \frac{Q-\mu}{\sigma}\right\}$$

Hence,

$$Z = \frac{Q-\mu}{\sigma}$$

Therefore, the optimal quantity to be ordered is;

$$Q^* = Z\sigma + \mu \qquad (6)$$

Where μ is the expected demand and σ is the standard deviation of demand.

(a) $If\ C_1 > C_0 \Rightarrow F(Q) = \frac{C_1}{C_1 + C_0} > 0.5;\ Z > 0.\ \ Hence, Q^* > \mu$

That is, if shortage cost is higher than excess inventory cost, order more than expected demand.

(b) If excess inventory is more costly than shortage cost, order less than demand;

$$C_1 < C_0 \Rightarrow F(Q) = \frac{C_1}{C_1 + C_0} < 0.5;\ Z < 0.\ \ Hence, Q^* < \mu$$

(c) If excess inventory and shortages cost are the same, order expected demand;

$$C_1 = C_0 \Rightarrow F(Q) = \frac{C_1}{C_1 + C_0} = 0.5;\ Z = 0.\ \ Hence, Q^* = \mu$$

Therefore, safety stock = Zσ.

The optimal probability of no shortage is;

$$P(D \le Q^*) = \frac{C_1}{C_1 + C_0} \qquad (7)$$

And the optimal probability of shortage is;

$$P(D > Q^*) = 1 - \frac{C_1}{C_1 + C_0} \qquad (8)$$

ProbabilityInventory Model for Demand with Weibull Distribution

The probability density function of a two parameter Weibull random variable is;

$$f(D;\lambda,k) = \begin{cases} \left(\frac{k}{\lambda}\right)\left(\frac{D}{\lambda}\right)^{k-1} e^{-\left(\frac{D}{\lambda}\right)^k}, D > 0 \\ 0, \qquad if \quad D \le 0 \end{cases} \qquad (9)$$

Where D =demand, is the random variable, $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter of the distribution.

The expected cost of inventory is given by;

$$E[C(Q,D)] = \int_0^\infty C(Q,D) f(D) dD$$

Where $f(D)$is the probability distribution of demand.

$$f(Q) = \int_0^\infty \frac{k}{\lambda}\left(\frac{Q}{\lambda}\right)^{k-1} e^{-\left(\frac{Q}{\lambda}\right)^k} dQ \tag{10}$$

andthe cumulative distribution function is;

$$F(Q) = 1 - e^{-\left(\frac{Q}{\lambda}\right)^k}$$

The optimal $Q^*$from (7) is given by;

$$1-e^{-\left(\frac{Q^*}{\lambda}\right)^k} = \frac{C_1}{C_1+C_0}$$

If $k = 1$; the rate of decrease in demand is constant with time.

If $k < 1$: the rate of decrease in demand decreases with time.

$k > 1$; if the rate of decrease in demand increases with time.

Assume that $k = 2$;

$$e^{-\left(\frac{Q^*}{\lambda}\right)^2} = 1 - \frac{C_1}{C_1+C_0} \tag{11}$$

$$\Rightarrow -\left(\frac{Q^*}{\lambda}\right)^2 = ln\left(1 - \frac{C_1}{C_1+C_0}\right)$$

Where $\lambda^2 = \frac{1}{n}\sum\sum_{i=1}^{n} D_i^2$

$$\lambda = \sqrt{\frac{\sum_{i=1}^{n} D_i^2}{n}}$$

The hypothesis regarding the distributional form is rejected at α level of significance, if the test statistic is greater than the critical value defined by$\chi^2_{1-\alpha,k-1}$, where k is the number of groups.

ANALYSIS AND RESULT

Shapiro-Wilk Test for Normality in the Distribution of DemandRate of Products

The demand rate in Table 1 for four different products which are Biscuit, Noodles, Drinks and Okpop (Sweet and Gum) were collected between January, 2014 and December, 2014.

Table1: Shapiro-Wilk test for normality in the distribution of demand

Products	Average demand($\bar{X}$)	Shapiro-Wilk Test Statistic	P-value
Biscuit (Cartons)	2741.67	0.940	0.503
Noodles (Cartons)	2683.33	0.948	0.609
Drinks (Packs)	1425.17	0.918	0.273
Okpop (Packs)	360.00	0.936	0.011*

Table 1 shows that the average demand for Biscuit is 2741.67cartons while that of Super pack noodles, Lacasera drink and Sweet and gum Okpop are 2683.33, 1425.17cartons, 1425.17packs and 360.00packs respectively. This record reveals that the average demand for Biscuit was higher than the average demand for other products. Also, since the distribution of the demand of products is not known, their distributions were assumed to be normally distributed. The null hypothesis is that the distribution of the demand is normal. In Table 1, the p-values of 0.503, 0.609, 0.273 and 0.011 were obtained for Blue Cabin Biscuits, Super pack Noodles, Lacasera drinks and Okpop Sweet and Gum respectively. Since the p-values are greater than 0.05 ($p >$

0.05) except for Okpop with $p = 0.011$. This means that the distribution of demand of these products are not different from normal distribution, except that of Okpop.

Inventory Planning for Blue Cabin Biscuit

Recall the probability of no shortage in (7) and given:$C1$=₦10 and $C0$=₦7;

$F(D \leq Q) = 0.588$

$P(D \leq Q) = P(D - \mu \leq Q - \mu)$=$P\left(\frac{D-\mu}{\sigma} \leq Z \leq \frac{Q-\mu}{\sigma}\right) = P(Z \leq z) = \sigma Z + \mu$

The value of Z can be obtained from the standard Normal curve as $Z = 0.222$

The optimal Quantity of Biscuit to be ordered is given by;$Q^* = \sigma Z + \mu$ = 2742

$\therefore Q^* = 3051\, units$ with safety stock =$\sigma Z = 309\, units$. The mean demand is 2742 with ± 3 standard deviation of 1392.Optimal probability of no shortage for Biscuit is; $P(D \leq 3051) = 0.588$ and optimal probability of shortage is; $P(D > 3051) = 0.412$.

InventoryPlanning for Noodles (Super Pack) and Lacasera Drink

Similarly, given:C_1(Shortage cost) =₦20 and C_0(excess inventory cost) =₦ 12.50 for Noodles and C_1 =₦16.5 and C_0 =₦ 9.5 for Lacasera, the optimal quantity, shortage and no shortage probabilities, safety stock, excess and shortage inventories costs/unit were obtained for these products. The results are summarized in Table 2.

Inventory Planning for Okpop (Sweet and Gum)

Since the demand for Okpop had failed the Shapiro-Wilk test for normality, the Chi-Squared goodness-of-fit test showed that the demand which decreases with time follows Weibull distribution using EasyFit software version 5.6. It was also used to estimate the shape and scale parameters of the distribution as follows: $\lambda = 336.7,\ k = 1.987$.Given: $C_1 = 8, C_0 = 2.50, k = 1.987, \lambda = 336.7$

From equation (11): $Q^* = 243.23 \approx 243$. Also, probability of shortage; $P(D \geq 295) = 0.762$ and probability of no shortage =$P(D < 295) = 1 - P(D \geq 295) = 0.238$

Table 2: Summary of Inventory Plan for Selected Products

Products	Optimal Probabilities		Optimal Quantity Q^*	Safety Stocks	Excess Inventory (C_0) (N)/unit	Shortag e Cost (C_1) (N)/unit
	Shortage	No Shortage				
Biscuit (Blue Cabin)	0.59	0.41	3051	309	7.00	10.00
Noodles(Super Pack)	0.62	0.38	2973	290	12.50	20.00
Drinks (Lacasera)	0.63	0.37	1530	105	9.50	16.50
Okpop (Sweet and Gum)	0.76	0.24	243		2.50	8

DISCUSSION

The results of the inventory analysis of products at a wholesale shop in Uyo, Nigeria showed that 3051 cartons of Blue Cabin Biscuit, 2973 cartons of Super Pack Noodles, 1425 packs of Lacasera Drinks and 243 packs of Okpop Sweet and Gum should be ordered monthly to reduce shortage and excess inventory. A safety stock of 309, 290 and 105 cartons/packs should be kept respectively for Blue cabin biscuit, Noodles, Lacasera drinks and Okpop Sweet and Gum to address unforeseen circumstances such as changes in customers' demand and also maximize profit. Also, the values of the probability of shortage and no shortage for the products respectively were: 0.41, 0.38, 0.37, 0.24 and 0.59, 0.62, 0.63, 0.76 for Blue Cabin Biscuit, Super Pack Noodles, Lacasera drink and Okpop Sweet and Gum. These depict the storage limits for the products and adherence to these values would help to increase the total profit of the business by preventing and minimizing excesses of stock by the company.

CONCLUSION

Demand of Blue Cabin Biscuit, Noodles and Lacasera drink have been shown to follow normal distribution while the demand of Okpop (Gum and Sweet) follows the Weibull distribution. Optimal order quantities and the probabilities of shortage and no shortage were obtained for the selected products over the planning horizon using their respective inventory models based on the probability distribution of demand as a variable.

REFERENCES

Basin, W.M. (1990). A Technique for Applying EOQ model to Retail Cycle Stock Inventory. *Journal of Small Business Management*, 28(1): 48-55.

Bloomberg, D., LeMay, S. and Hanna, J. (2002). *Logistics*. Upper Saddle River: Prentice Hall: 200-260.

Coyle, J.J.: Bardi, E. J and Langley, J.C. (2003). *The Management of Business Logistics: A Supply Chain Perspective.* Canada: Thomson South-Western: 453-560.

Drury, C. (1996).*Management and cost Accounting, London*: International Housan Business Press: 632-646.

Fuerst, W.L. (1981).Small Business Get A New Look at ABC Analysis for Inventory Control. *Journal of Small Business managements*, 19(3): 39-44.

Goldsby, T. and Martichenko, R. (2005). *Lean Six Sigma Logistics.* Boca Raton: J. Ross Publishing. Inc.: 71-83.

Hwang, H., Hahn, K.H (2000). An-optimal procurement policy for items with an inventory level dependent demand rate and fixed life time. *European Journal of Operation Research,* 127(3):537-545.

Hycinth C. I., H. C., Ogbonna, C. J. Ogbonna, Opara, J., Onuma, K. G. (2014). Application of Inventory Model in Determining Stock Control in an Organization. *American Journal of Applied Mathematics and Statistics* 2(5) (2014): 307-317.

Kolter P. (2000). *Marketing Management,* 2nd Edition. The Millennium Edition. New Delhi: Prentice Hill of India: 355-456.

Lambert, D. J. Cooper, M.C and Pagh, T.D. (1998). Supply chain management, implementation issues and research opportunities. *International Journal of Logistics Management,* 9(2):1-19.

Minner, S. (2000).*Strategic safety stocks in supply chains*. New York: Springer, Heidelberg, pp 77-178.

Morris C. (1995). *Quantitative Approaches in Business Studies*: 7th Edition, London: Pitman Publisher, pp 125-235.

Nahmias, S. (1997). *Production and operation analysis*. New York: McGraw-Hill International, 6th Edition, pp175-245.

Rosenblatt, B.S., (1977). *Modern Business*. *A Systems Approach* (2nd Edition). Bostion: Houghton Mifflin Co., pp 345-350

Schroeder, R. G. (2000). Operations Management, Contemporary *Concepts and Cases*. USA: International Edition, pp 175-290.

Simchi-Levi, D., Kaminsky, P., and Simchi-Levi, E. (2004).*Managing The Supply Chain*: The Definitive Guide For The Business Professional. New York: McGraw-Hill. : 13-22.

Toomey, J.W. (2000). *Inventory Management: Principles, Concepts and Techniques*. Norwell; Kluwer Academic Publishers. http://dx.doi.org/10.1007/978-1-4615-4363-3, pp 116-168.