# VALIDITY OF THE HARMONIC ANALYSIS OF MONTHLY RAINFALL VARIABILITY IN MAKURDI, NIGERIA

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**ABSTRACT:** This work investigates the effectiveness of the harmonic analysis as a tool in the estimation of annual rain amounts in Makurdi, Nigeria between the years 1977-2008. The validation of the estimated rainfall values of the years 2011 – 2014 was done using a T- test analysis on the estimated and measured rainfall values of 2009 and 2010 data. The results show that rainfall variability in Makurdi, Nigeria is strongly periodic and the cyclic fluctuations in the rainfall are dominated by first harmonics. The large amplitude (89.55 %) of the first harmonic indicates strong annual variation of rainfall in the station. The mean monthly rainfall over the station shows a bimodal pattern of rainfall with the peaks in June and September. The intensity or dearth of average rainfall in a day/month depends on previous event. The 2009-2010 prediction of monthly average rainfall for the station was validated at 95% confidence level. Hence, the harmonic analysis predictions for the four other years ahead are valid as far as rainfall variations in Makurdi are concerned.

## INTRODUCTION

Understanding of the spatial and temporal patterns of the rainfall variations is essential for the knowledge of water balance dynamics on various scales, and is useful for water resource management and planning. Harmonic Analysis, which is commonly applied to study periodic variations, decomposes a time series into its constituent parts if the time series represents a periodic phenomenon. It transforms a complex time series to a sum of many sinusoidal functions or harmonics (Park, 2008).

The Harmonic analysis is based on a mathematical principle that a curve, viewed as a function, may be represented by a series of trigonometric functions. Hence, the series formula is given by Panofsky and Brier (1960):

$$X_t = \bar{X} + \sum_{i=1}^{\frac{N}{2}} [ A_i \sin(\frac{360}{p} it) + B_i \cos(\frac{360}{p} it)] \quad (1)$$

$$A_i = \frac{2}{N} \sum_{i=1}^{\frac{N}{2}} [ \bar{X} \sin(\frac{360}{p} it), B_i = \frac{2}{N} \sum_{i=1}^{\frac{N}{2}} [ \bar{X} \cos(\frac{360}{p} it)] \quad (2)$$

$X_t$ is the observed value at time, t, $\bar{X}$ is the arithmetic mean, $A_i$ and $B_i$ are coefficients or the amplitudes, N is the number of observations, i is the number of harmonics and P is the period of observation. In other words, the time series equals the mean plus the sum of all $\frac{N}{2}$ harmonics. The equation (1) above can be rewritten as;

$$X_t = \bar{X} + \sum_{i=1}^{\frac{N}{2}} C_i \cos[(\frac{360}{p} i)(t - t_i)] \quad (3)$$

The type of variation dominating the curve is revealed by a comparison of the sizes of the amplitudes $C_i$, where $C_i = \sqrt{A_i^2 + B_i^2}$ is the amplitude of the $i^{th}$ harmonic and $t_i = \frac{P}{360}$ arc sin ($A_i/C_i$) is the time at which the $i^{th}$ harmonic has a maximum. It can also be expressed in percentage. A harmonic with overwhelming contribution would definitely account for most of the periodic variation in the data, while the contributions of the other harmonics would be considered negligible. Hence, a large first harmonic amplitude suggests strong annual variation, while comparatively large second harmonic amplitude points to strong semiannual variation.

Harmonic analyses have been applied in the investigation of some climatic variables. Scott and Shulman, (1979) used it for areal and temporal analysis of rainfall. It has been used to study seasonal global precipitation by Hsu and Wallace, (1976). Hastenrath (1968) used it to study precipitation in Central America. Other studies of rainfall variation with harmonic analysis include: Studies of seasonal variation of rainfall (Kirkyla and Hammed, 1989; Kadioglu *et al.*, 1999; Vines, 2007 and Wong *et al.*, 2009). Isikwue *et al.*, (2011), investigated the use of harmonic analysis to study hourly temperature variability in Makurdi, Nigeria.

This work investigates the validity of the harmonic analysis model in the estimation of rainfall amounts in Makurdi, Nigeria. Makurdi is the capital city of Benue state located in the north-central of Nigeria. It lies on the south bank of the Benue River. It is located at latitude 7.74$^{o}$N, longitude 8.51$^{o}$E at an elevation of 104 m above sea level.

## METHODOLOGY

The daily rainfall data used in this study were obtained from the International Institute for Tropical Agriculture (IITA) Ibadan, Nigeria. The period of this record is thirty-two (32) years from 1977-2008. The rainfall data were daily data yielding a sample size of about 30 for each month. Harmonic analysis on the averaged monthly rainfall observations was performed using the Turbo Pascal for windows programming language in order to implement the computation. This was carried out by fitting a periodic function of sinusoidal character to enhance the determination of the contribution of each harmonic (expressed as a percentage of total variation in the rainfall measurements it accounts for), the amplitude of each harmonic and the time at which each harmonic is maximum.

Autocorrelation plots of the data (using SPSS statistical software) were made in order to determine whether significant dependence exists in the data.

Paired sample T-test (using the Statistical Package for Social Science, SPSS) was used to determine whether there is a significant relationship between the estimated rainfall values of the years 2008 and 2009 and their corresponding observed values. Hence, this is used to validate the efficiency of the harmonic analysis model in estimating the rain fall amount in Makurdi, Nigeria. Table 1 links the months to the corresponding year.

Table 1: Code for the Month axis along with the year

| Month | Year | Month | Year |
|---|---|---|---|
| 0-12 | 1977 | 204-216 | 1994 |
| 12-24 | 1978 | 216-228 | 1995 |
| 24-36 | 1979 | 228-240 | 1996 |
| 36-48 | 1980 | 240-252 | 1997 |
| 48-60 | 1981 | 252-264 | 1998 |
| 60-72 | 1982 | 264-276 | 1999 |
| 72-84 | 1983 | 276-288 | 2000 |
| 84-96 | 1984 | 288-300 | 2001 |
| 96-108 | 1985 | 300-312 | 2002 |
| 108-120 | 1986 | 312-324 | 2003 |
| 120-132 | 1987 | 324-336 | 2004 |
| 132-144 | 1988 | 336-348 | 2005 |
| 144-156 | 1989 | 348-360 | 2006 |
| 156-168 | 1990 | 360-372 | 2007 |
| 168-180 | 1991 | 372-384 | 2008 |
| 180-192 | 1992 | 384-396 | 2009 |
| 192-204 | 1993 | 396-408 | 2010 |

## ANALYSES / DISCUSSION

Figures 1 shows the time series plot of average monthly rainfall, while Figure 2 shows the harmonics plot of average monthly rainfall for Makurdi .

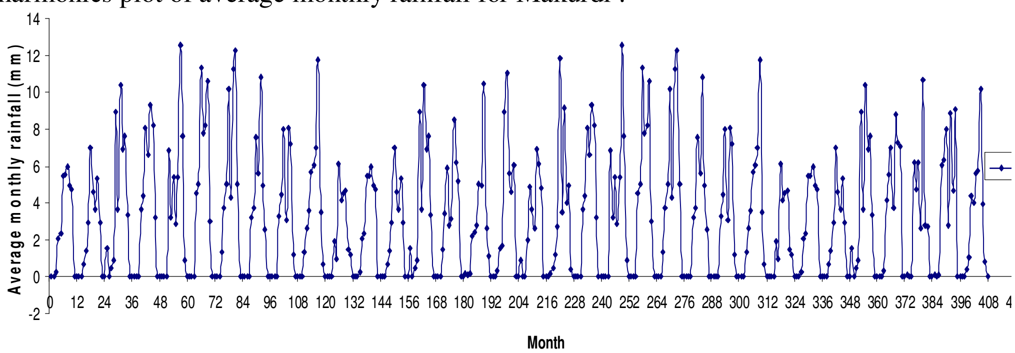


Figure 1: Time series plots of average monthly rainfall for Makurdi Station

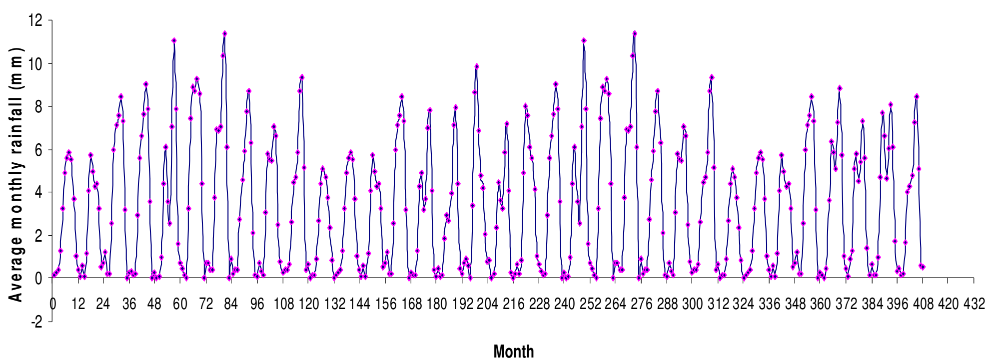


Figure 2: Harmonics plot of average monthly rainfall for Makurdi Station

**Periodic Variability and harmonic characteristics**

Both the time series and the harmonic plots (Figures 1and 2) of the rainfall data in the station indicate periodicity in the rainfall variations. This periodic variability could be explained by the seasonal movement of south-westerly air-mass from the Atlantic Ocean northwards between

January and October, and it retreat from the southern fringe of the Sahara desert when pushed back by the north-easterly air-mass from the desert (Okajima *et al.*, 2003). In addition, the presence of River Benue (one of the two largest rivers in Nigeria), that runs across the middle of Makurdi town might also have influence on the rainfall variation in the station. This is because; the river acts as a heat reservoir, which might contribute in the convective rainfall .in the station.

The time series curves exhibit unimodal and bimodal patterns of rainfall, having series of peaks in a particular year. However, for the harmonics plots, the bimodal patterns are only prominent in the years 1981; 1991; 1994; 1997; 2007 -2009. Probably, others were due to noise in the data which have been filtered out by the harmonic analysis. This observation of bimodal patterns of rainfall could imply its occurrence every three to ten years. However, in the last three years (2007 -2009), it occurred yearly. This could be attributed to changes in other climatic phenomena which are subject to further verification

The harmonic and time series plots of the mean monthly rainfall over the station (Fig. 3) show a bimodal pattern of rainfall with the peaks in June and September, while a break is observed in July. Little or no rain was observed in the months of November to February. The curves of harmonic and time series plots are out of phase in the months between March and August; but are in phase in the months between September and February.

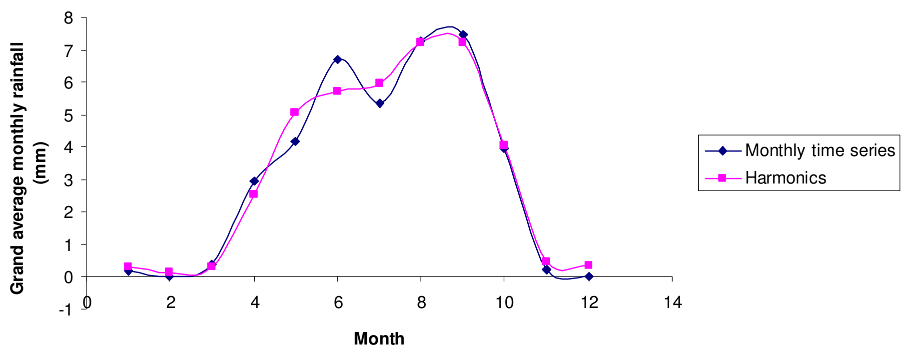


Figure 3: Mean monthly rain fall and harmonics plots for Makurdi Station

Result extract from the run of harmonic analysis on average monthly rainfall is summarized in Table 2. It also shows that the first harmonic dominates the periodic components in the monthly average rainfall of the station as it has the highest percentage contribution of 89.55%, indicating that the monthly rainfall in the station actually fluctuates. The large amplitude of the first harmonic indicates strong annual variation of rainfall in the station (Wong *et al.*, 2009 and Kirkyla and Hammed, 1989). This is also evident in Figure 3, where the months of June and September are prominent in the annual rainfall fluctuations.

**Estimation and Prediction**

Using the average monthly rainfall obtained as 3.22mm, the period of 12 months, and the sine and cosine coefficients in Table 2, the periodic function $X_t$ for the monthly average rainfall of Makurdi was obtained using extracts from equation (1) as:

$$X_t = 3.22 + \sum_{i=1}^{3} [A_i \sin(30\,it) + B_i \cos(30\,it)] \quad (4)$$

Where $A_i$ and $B_i$ are coefficients of sine and cosine respectively and i's are integers ranging from 1 to 3 as given in Table 2. Equation (4) was then implemented in the harmonic analysis program so as to make a six year forecast of monthly average rainfall measurements for Makurdi.

Table 2: Result extract from the run of harmonic analysis program on average monthly rainfall of Makurdi, Nigeria.

| Harmonics | Sine Coefficients ($A_i$) | | | Cosine Coefficients ($B_i$) | | | Amplitude | Percentage Contribution % |
|---|---|---|---|---|---|---|---|---|
| | A1 | A2 | A3 | B1 | B2 | B3 | | |
| 1st | -2.4867 | 0 | 0 | -2.9550 | 0 | 0 | 3.86 | 89.55 |
| 2nd | 0 | 0.2186 | 0 | 0 | -0.5486 | 0 | 0.59 | 2.09 |
| 3rd | 0 | 0 | 0.9746 | 0 | 0 | -0.0746 | 0.98 | 5.74 |

Observations show that equation 4 exhibits a good fit to the average monthly rainfall as it produced very close estimates of the actual monthly average rainfall. It yielded the same mean (average monthly rainfall) as that of the actual data; and very close standard deviation of the actual and that of the model estimates as shown in Table 3, which also displays the result of the forecast with the corresponding actual and estimates of monthly average rainfall measurements. These estimates are known as model estimates. Although the standard deviations are high compared to the mean values, this could be explained by considering the variability inherent in the original data.

Table 3: Six years (2009 – 2013) forecasts of average monthly rainfall for Makurdi.

| Month | Actual Value (mm) | Model Estimate (mm) | Predictions for Six years 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| January | 0.178629 | 0.322660 | 0.322846 | 0.323062 | 0.323277 | 0.323491 | 0.323706 | 0.323919 |
| February | 0.008460 | 0.139803 | 0.139610 | 0.139342 | 0.139074 | 0.138806 | 0.138538 | 0.138270 |
| March | 0.386290 | 0.309277 | 0.309676 | 0.310362 | 0.311049 | 0.311736 | 0.312425 | 0.313114 |
| April | 2.909375 | 2.521540 | 2.522280 | 2.523953 | 2.525627 | 2.527300 | 2.528973 | 2.530647 |
| May | 4.090524 | 5.004422 | 5.004715 | 5.005682 | 5.006649 | 5.007615 | 5.008580 | 5.009544 |
| June | 6.753854 | 5.721880 | 5.721888 | 5.721938 | 5.721987 | 5.722036 | 5.722085 | 5.722134 |
| July | 5.431855 | 6.047009 | 6.047022 | 6.047528 | 6.048034 | 6.048541 | 6.049048 | 6.049556 |
| August | 7.285685 | 7.283301 | 7.283225 | 7.283883 | 7.284541 | 7.285198 | 7.285855 | 7.286511 |
| September | 7.486146 | 7.168549 | 7.168808 | 7.167790 | 7.166771 | 7.165751 | 7.164730 | 7.163708 |
| October | 3.802722 | 3.958162 | 3.959105 | 3.956711 | 3.954316 | 3.951921 | 3.949526 | 3.947131 |
| November | 0.221979 | 0.423656 | 0.424380 | 0.423023 | 0.421667 | 0.420313 | 0.418959 | 0.417607 |
| December | 0.001613 | 0.348090 | 0.348279 | 0.347997 | 0.347715 | 0.347432 | 0.347148 | 0.346864 |
| Monthly Average | 3.21 | 3.21 | | | | | | |
| Standard deviation | 3.02 | 2.85 | | | | | | |

Table 4 displays both the measured and the estimated rainfall values in Makurdi for the years 2009 and 2010. The level of deviation measured from the estimated rainfall values in both years were determined using Paired sample T-test analysis. The result shows that there is no significant difference at 95% confidence level between the measured and the estimated rainfall in both years as can be seen from the T and P values. Hence, the predictions for the four other years ahead are valid as far as rainfall variations in Makurdi are concerned.

SPSS Autocorrelation plot (Figure 4) shows a graphical distribution of autocorrelation values of the monthly average rainfall of Makurdi station and their corresponding lags in a confidence band. Two facts are evident in Figure 4, there are persistent patterns of rainfall variations as shown by the consistent deviations of the actual measurements from the standardized mean, and secondly, the presence of outliers of the confidence band indicates that the monthly rainfall in the station is not independent. That is the intensity or dearth of rainfall in a year could have a spillover effect on the next year or from previous year as the case may be. This could be the effect or the result of the periodic nature of rain events in the station.

Table 4: Validation results for Makurdi station

| | 2009 | | 2010 | |
|---|---|---|---|---|
| Month | Actual Value | Model Estimate | Actual Value | Model Estimate |
| January | 0.074194 | 0.322846 | 0.000000 | 0.323062 |
| February | 0.000000 | 0.139610 | 0.000000 | 0.139342 |
| March | 0.096774 | 0.309676 | 0.406452 | 0.310362 |
| April | 6.003333 | 2.522280 | 1.046667 | 2.523953 |
| May | 6.332258 | 5.004715 | 4.345161 | 5.005682 |
| June | 7.990000 | 5.721888 | 3.980000 | 5.721938 |
| July | 2.745161 | 6.047022 | 5.570968 | 6.047528 |
| August | 8.880645 | 7.283225 | 5.745161 | 7.283883 |
| September | 4.683333 | 7.168808 | 10.186667 | 7.167790 |
| October | 9.109677 | 3.959105 | 3.912903 | 3.956711 |
| November | 0.040000 | 0.424380 | 0.800000 | 0.423023 |
| December | 0.000000 | 0.348279 | 0.000000 | 0.347997 |
| **Mean** | 3.829615 | 3.270986 | 2.999498 | 3.270939 |
| **Standard Deviation** | 3.7577108 | 2.906983 | 3.1877495 | 2.90704 |
| **T-value** | | 0.816 | | -0.759 |
| **P-value** | | 0.432 | | 0.464 |
| **Remarks** | No significant | Difference | No significant | Difference |

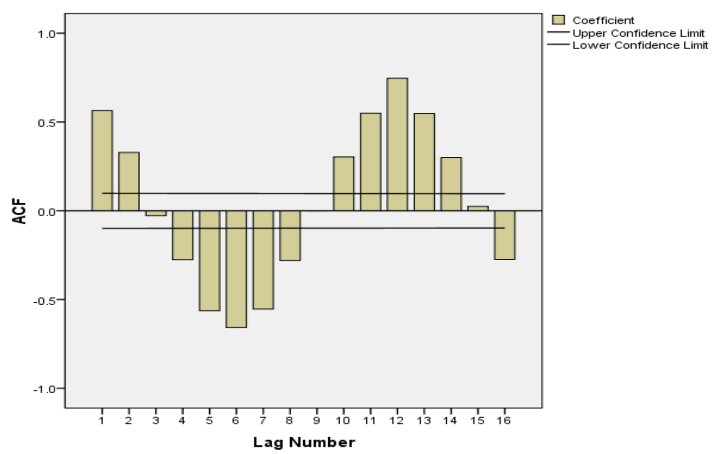


Figure 4: Autocorrelation plot of average monthly rainfall for Makurdi.

## CONCLUSIONS

The following conclusions were drawn from the analyses:

- Rainfall variability in Makurdi, Nigeria is strongly periodic. Its periodicity can be determined using harmonic analysis model. Rainfall variability in Makurdi is caused by the movement of south-westerly air-mass from the Atlantic Ocean northwards between January and October, and its retreat from the southern fringe of the Sahara desert when pushed back by the north-easterly air-mass from the desert.
- The cyclic fluctuations in the rainfall are dominated by first harmonics. The large amplitude (89.55%) of the first harmonic indicates strong annual variation of rainfall.
- The mean monthly rainfall over the station shows a bimodal pattern of rainfall with the peaks in June and September, while a break is observed in July. Little or no rain was observed in the months of November to February.

- The intensity or dearth of average rainfall in a day/month depends on previous event.
- The presence of such features as large water body identified in this station might have some influence on the variability of monthly rainfall.
- The 2009-2010 prediction of monthly average rainfall for the station was validated by the t-test analysis at 95% confidence level. Hence, the harmonic analysis predictions for the four other years ahead are valid for rainfall variations in Makurdi.

## RECOMMENDATIONS

- The harmonic model could be applied to the rainfall data of other stationsto obtain the fluctuations and patterns of rainfall in those stations and Nigeria at large.
- This model can be applied to other atmospheric variables (of the same period), such as air temperatures. The comparison of such results will enhance determination of the climatic variability in Nigeria.
- Further research could be advanced to determine the reason for the lag between the harmonics and time series plots
- The influence of other atmospheric phenomena such as the El- Nino Southern Oscillations could be investigated in order to determine they have any effect on the rainfall pattern.

## ACKNOWLEDGMENTS

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