MATHEMATICAL SCIENCE AND TECHNOLOGICAL / INDUSTRIAL DEVELOPMENT OF A MODERN SOCIETY

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ABSTRACT: The role of mathematical science in enhancing manpower development for effective take-off of technological and industrial development of a country such as ours is examined. In this paper, we employ two models; mathematical and statistical to discuss certain national problems such as social and economic problems with a view to solving them. Finally, the interrelationship of mathematics with other science based disciplines in fostering scientific research is highlighted.

INTRODUCTION

The use of mathematical science in solving real-world problems has become widespread in recent times. This is partly due to the use of the systems approach to problem solving, and to the increasing computational power of digital computers and computing methodology, both of which have made many problems amenable to mathematical treatment. There is hardly a branch of learning where mathematics and computing have not made an impact. The use of mathematics is one of many approaches to solving real world problems. Others include experimentation either with scaled physical models or with the real world directly.

For instance, Bestman (1984) modeled flow of blood in arteries as viscous flow in cylindrical tubes, while Chow (1959) discussed extensively hydraulics of open channel flow. Eyo (2007), Gill (1980), among others also contributed to the discussion of flow along open channels. In this paper, Eyo (2007) used the method of best hydraulic performance to discuss the excavation of a rectangular open channel for optimum discharge. Also, Poisson distribution (Harper, 1977) was used to describe queuing problem such as arrival at a GSM sim registration centre. Finally, the paper discusses the role of mathematics in the development of science and technology particularly in developing countries.

Steps in Mathematical Modeling

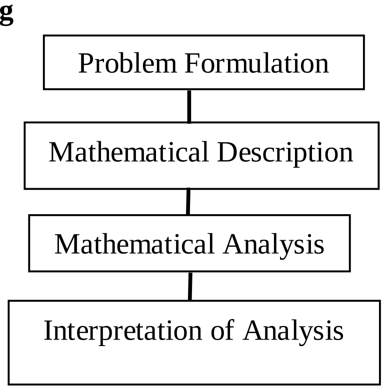

Figure 1. Modeling steps

The steps involved in using mathematics to solve real-world problems are shown in Figure 1. The most crucial and important step is the satisfactory translation of the problem from the real physical world into a mathematical description. Once this is done, standard techniques of mathematical analysis can be used to obtain a solution to the problem. The validity of the solution depends on how well the mathematical description models the real world.

Social Problem: Provision of a GSM Sim Registration Centre (Case Study One)
The GSM company will like to promote her business in terms of security of her services to her customers by increasing the sim registration points when convinced that an arrival would have to wait a long time for the registration. The problem facing the company is to determine by how much the flow of arrivals must be increased in order to justify more registration points. The paper attempts to build a mathematical model to help solve this problem.

The problem is statistical in nature as arrivals at a sim registration point are considered to be Poisson. We illustrate this with the provision of one registration point and then observe how justifiable it will be for the company to provide a second registration point. The validity of the model will then warrant the provision of a second or more registration centres for optimum profit by the company.

Development of Mathematical Model

Let λ = 0.1 arrival per minute
μ = 0.33 service per minute (fixed)
P = Probability of an event occurring

Therefore

(i) P{an arrival has to wait} $= 1 - P_0 = \dfrac{\lambda}{\mu}$ (1)

or
$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad (2)$$

where n = non-negative integer. When n = 0, (2) yields (1). So (1) is a special case of (2).

(ii)
$$E(m/n > 0) = \frac{\mu}{\mu - \lambda} \quad (3)$$

where $E(m/n > 0) =$ average length of the queues that form from time to time

(iii)
$$E(w) = \frac{\lambda}{\mu(\mu - \lambda)} \quad (4)$$

where *E(w)* = waiting time of an arrival.

Numerical Example

Arrivals at a sim registration centre are considered to be Poisson, with an average time of say 10 minutes between one arrival and the next. The length of the registration is assumed to be distributed exponentially, with mean 3 minutes. We shall develop a model to determine

(i) the probability that a person arriving at the centre will have to wait.
(ii) the average length of the queues that form from time to time, and
(iii) whether the company is justified to provide a second or more centres.

Solution

(i) Substituting the values of λ and μ in (1), we find $P_0 = \dfrac{0.23}{0.33}$, so that

$$P\ \{\text{an arrival has to wait}\} = 1 - \frac{0.23}{0.33} = 0.3$$

(ii) Similar substitution of the values of λ and μ in the model (3) gives $\dfrac{0.33}{0.23} = 1.43$ persons.

(iii) Finally, we will seek the new value λ', say for which *E(w)=3* minutes. Thus using the model (4), we find
$$3 = \frac{\lambda'}{0.33(0.33 - \lambda')}$$

Solving for λ' we obtain $\lambda' = 0.16$ arrival per minute. This yields approximately 10 arrivals per hour while $\lambda = 0.1$ arrival per minute gives 6 arrivals per hour. Thus with the new value λ', the GSM company is justified to provide a second centre or more in order to alleviate the suffering (social problem) of the people and also maximize profit.

Economic Problem: Dredging of Open Channel to Boost Navigation (Case Study Two)

The problem confronting navigators is that many open channels (e.g. rivers) cannot permit big ships to enter into the harbour owing to the shallowness of water. If the water level is increased through excavation this problem will have been solved.

Mathematical Model for Rectangular Open Channel

For purpose of illustration, we shall develop a mathematical model to excavate a rectangular open channel.

Model for Original Rectangular Channel Section

Cross sectional area A $A = b_0 y_0$ (5)

where b_0 and y_0 are the width and depth of the original channel respectively.

Wetted Perimeter p_0, $p_0 = b_0 + 2y_0$ (6)

From (1), $b_0 = \dfrac{A}{y_0}$ (7)

Substituting (7) in (6), we find $p_0 = \dfrac{A}{y_0} + 2y_0$ (8)

Hydraulic mean depth m_0 $m_0 = \dfrac{A}{y_0} = \dfrac{b_0 y_0}{b_0 + 2y_0}$ (9)

Discharge Q_0. From Manning's formula (Chow 1959)

$$Q_0 = \frac{1}{n} A m_0^{\frac{2}{3}} S_0^{\frac{1}{2}} \quad (10)$$

where n = roughness factor, S_0 = bed slope of the channel.

Mean Velocity u_0 $u_0 = \dfrac{Q_0}{A}$ (11)

Froude number F_0 $F_o = \dfrac{u_0}{\sqrt{g y_0}}$ (12)

For best hydraulic performance the wetted perimeter must be a minimum. Thus differentiating (8) with respect to y_0 and setting it equal to zero, we find

$$\frac{dp_0}{dy_0} = A y_0^{-2} + 2 = 0$$

giving $A = 2y_0^2$ (13)

Substituting (1) in (2) we obtain $y_0 = \dfrac{b_0}{2}$ (14)

as a condition for hydraulic effectiveness.

Model for New (Excavated) Rectangular Channel Section

New depth of the channel y_N

Since the cross sectional area must be unchanged, then from (13),

$$y_N = \left(\frac{A}{2}\right)^{\frac{1}{2}} \tag{15}$$

By continuity $b_0 y_0 = A = b_N y_N$ (16)

New width of the Channel b_N

Substituting (15) in (16) we find $b_N = (2A)^{\frac{1}{2}}$ (17)

New Wetted Perimeter p_N, $p_N = b_N + 2y_N$ (18)

New Hydraulic Mean Depth m_N

$$m_N = \frac{b_N y_N}{b_N + 2y_N} \tag{19}$$

New Discharge Q_N

Again, (Chow 1959) $Q_N = \frac{1}{n} A\, m_N^{\frac{2}{3}} S_0^{\frac{1}{2}}$ (20)

New Mean Velocity u_N $u_N = \frac{Q_N}{A}$ (21)

New Froude number F_N $F_N = \frac{u_N}{\sqrt{g y_N}}$ (22)

Numerical Illustration

A rectangular open channel 6.5m wide and 1.2m deep has a slope of 1 in 1000 and is lined with rubble masonry (Manning's n = 0.017). We wish to increase the amount of water discharged as much as possible through excavation without changing the channel slope or the rectangular form of the section. Using the model above we will determine (i) the discharge of the original channel (ii) new dimensions of the channel to give maximum discharge, and (iii) the new discharge.

Solution

Original Channel

Here $b_0 = 6.5m$, $y_0 = 1.2m$, $S_0 = \frac{1}{1000}$

Substituting these data in the model (5), (6) and (9) we find

$$A = 7.8m^2, \quad p_0 = 8.9m, \quad m_0 = 0.876,$$

while similar substitution in the model (10), (11) and (12) gives

$$Q_0 = 13.28m^3/s, \quad u_0 = 1.7m/s, \quad F_o = 0.4954$$

New Channel

Using the above data appropriately in the model (15), (17), (18), (19), (20), (21) and (22) we obtain respectively

$$y_N = 1.97m, \quad b_N = 3.94m, \quad p_N = 7.88m$$

$$m_N = 0.989m, \quad Q_N = 14.4m^3/s, \quad u_N = 1.84m/s, \quad F_N = 0.4185$$

RESULTS

The result of the case studies one and two are:

Cast Study One

(i) Probability that an arrival has to wait = 0.3

(ii) Average queue length = 1.43

(iii) $\lambda' = 0.16$ yields 10 arrivals per hour against $\lambda = 0.1$ which gives 6 arrivals per hour. Thus, with the increase in arrivals the company can provide a second or more registration centres.

Case Study Two

Characteristics	Original Channel	New Channel
Bed slope S_0	$\frac{1}{1000}$	$\frac{1}{1000}$
Manning's *n*	0.017	0.017
Area of cross section A	$7.8m^2$	$7.8m^2$
Width b	6.5m	3.94m
Depth y	1.2m	1.97m
Wetted perimeter P	8.9m	7.88m
Hydraulic mean depth	0.876m	0.989m
Discharge Q	$13.28m^3/s$	$14.4m^3/s$
Mean velocity u	1.7m/s	1.84m/s
Froude number	0.4954	0.4185

Mathematics and Its Role in other Disciplines

Medical scientists use mathematics to model the flow of blood in arteries as viscous flow in cylindrical tubes (Bestman 1984). In chemistry, determination of the rate of chemical reactions, molecular weight of a substance and balancing of chemical equations etc. couldn't be possible without the knowledge of mathematics.

The study of laws of light, pendulum, electricity etc., in physics is made possible through mathematics. For instance, partial and ordinary differential equations arising from physics can only be solved by using appropriate mathematical techniques. Biologists use it to model seasonal plant growth in order to be able to predict the behaviour of the size of the plant at the initial and later times. Statisticians use it to discuss randomness of a given set of data, solve queuing problems as well as make prediction of certain phenomena from basic data.

In technological field, the role of mathematics cannot be over-emphasized for instance, aeronautical engineers use it to calculate the lift and drag forces on aircrafts. Marine engineers need it to discuss the motion of ships through oceans. Civil engineers use it to discuss the motion of water in rivers, canals, civil water supply systems and sewage channels. Chemical engineers need it to discuss mass, momentum and heat transport in chemical fluid flowing in pipes and channels, while mechanical engineers apply it to explain mass and heat transport in machines.

Finally, other areas like control of ecology, urban planning, weather forecasting, oil exploration, water resources, agriculture, transportation, banking and insurance all have mathematics component.

CONCLUSION

In case study one, the result of the analysis reveals that the company is justified to provide a second registration point or more to alleviate social problem, whereas in case study two, the high level water or increase in discharge of the channel will allow big ships to sail to the harbour without grounding.

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