ON THE GENERALIZED EQUATION OF A HOLLOW ANODE CASCADING PLASMA FOCUS

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ABSTRACT

Shadow graphic studies of plasma dynamics in the plasma focus reveal that targets placed downstream of the anode become auxiliary anode to form another focus event, thus instead of the familiar three phases i.e. Axial phase (A), Radial phase (R) and Radial Extension phase (RA) there are six phases altogether. A model for two targets with holes at the center placed downstream has been proposed resulting in nine phases, but the equations are unwieldy and rather laborious. In this paper, the equation have been written in generalized form in terms of n so that an equation applicable in the first axial phase (nA, n = 1) is also applicable in the second axial phase (nA, n = 2) and the third axial phase (n A, n = 3) by inputting n = 1, 2, 3 successively in a given equation. The same is true for the radial phases (n R, n = 1, 2, 3) and the radial extension phases (n C, n = 1, 2, 3).

INTRODUCTION

Metals and deuteriated (deuterium impregnated metal) targets have been used at various axial distances; Moo *et al* (1991) and Lee *et al* (1988) used a 3KJ UNU/ICTP plasma focus to measure neutron production The result showed that about 85% of the neutrons are due to deuteron beam-gas target mechanism and they were careful not to place the target closer than 2cm of a focus with anode radius 1cm, being conscious of the fact that current and voltage measurement showed the dynamics is not affected when the target is placed at position farther than 2cm. Lee (1991) set up a two solid targets to produce three cascading focus events, with each event made up of an axial run down phase (A) a radial collapse phase (R) and a radial extension phase (RA) resulting in nine phases. In his model, he explained the possibility of a cascading focus with characteristics of good focusing achieved. Based on the work of Lee (1991), Alabraba (2015) set up a model for two hollow anodes instead of solid anodes as shown in Fig. 1 with a view of increasing the neutron yield with deuterium gas. However the equations involved in these nine phases is rather laborious and so in this paper the method of mathematical induction is employed to write the equations in generalized form.

FORMULATION

The 9-phase hollow anode cascading focus has three phases in the axial phases (a,d,g) in Fig.1. Here a snowplow model and a circuit equation is used to find the current and position of the current sheath while in the radial phases (b, e, h), (Fig.1), the slug model of Lee(1989) and Potter(1978) have been used. In the radial extension phases(c, f, i) in Fig.1, shadow graphic result show that there is no visible column and so the model adopted is that the current path linking the anode to the moving current sheath is large and flows uniformly in a column. This is seen to extend through the holes of the cascade anode as shown in phase 4 i.e. d through to phase 9 i.e. i in Fig.1. In the axial and radial extension phases the snowplow equations require mass loading. The $\vec{J}X\vec{B}$ forces, the mass loading and inductances of the plasma incorporating the capacitor discharge of the plasma focus shown in Fig.2 are listed thus:

(1) Axial Phases

Inductance in Phase 1 (a in Fig.1)

$$L_{p1} = \frac{\mu_0}{2\pi} z \ln c \quad 1)$$

Inductance in phase 4 (d in Fig.1)

$$L_{p4} = \frac{\mu_0}{2\pi}\left[(\ln c)z_0 + \ln c_1(z - z_0) + \frac{1}{2}(z_1 - z_0) + \frac{1}{2}(z - z_1)d_{11}^2\right] \quad 2)$$

Inductance in phase 7 (g in Fig 1)

$$L_{p7} = \frac{\mu_0}{2\pi}\left[(\ln c)z_0 + \ln c_1(z_2 - z_0) + lnc_2(z - z_2) + ½(z_1 - z_0 + z_3 - z_2) + \frac{1}{2}(z_2 - z_1)d_{11}^2 + \frac{1}{2}(z - z_3)d_{22}^2\right] \quad 3)$$

Mass loading in Phase 1

$$m_1 = \pi\rho_0 a^2(c^2 - 1)z \quad 4)$$

Mass loading in Phase 4

$$m_4 = \pi\rho_0 a^2\left[(c^2 - 1)z_0 + (z - z_0)\left(c^2 + d_1^2(d_{11}^2 - 1)\right)\right] \quad 5)$$

Mass loading in Phase 7

$$m_7 = \pi\rho_0 a^2\left[(c^2 - 1)z_0 + (z_2 - z_0)\left(c^2 + d_1^2(d_{11}^2 - 1)\right) + (z - z_2)\left(c^2 + d_2^2(d_{22}^2 - 1)\right)\right] \quad 6)$$

Axial force in Phase 1

$$F_1 = \frac{\mu_0}{4\pi}(lnc)i^2 \quad 7)$$

Axial force in Phase 4

$$F_4 = \frac{\mu_0}{4\pi}\left(lnc_1 + \frac{d_{11}^4}{4}\right)i^2 \quad 8)$$

Axial force in Phase 7

$$F_7 = \frac{\mu_0}{4\pi}\left(lnc_2 + \frac{d_{22}^4}{4}\right)i^2 \quad 9)$$

(2) Radial Phases

Inductance in Phase 2 (b in Fig. 1)

$$L_{p2} = \frac{\mu_0}{2\pi}\left[(lnc)z_0 + ln\left(\frac{b}{r_p}\right)z_f\right] \quad 10)$$

Inductance in Phase 5 (e in Fig.1)

$$L_{p5} = \frac{\mu_0}{2\pi}\left[(lnc)z_0 + lnc_1(z_2 - z_0) + \frac{(z_1 - z_0)}{2} + ln\left(\frac{b}{r_p}\right)(z - z_2) + \frac{(z_2 - z_1)d_{11}^2}{2}\right] \quad 11)$$

Inductance in Phase 8 (h in Fig. 1)

$$L_{p8} = \frac{\mu_0}{2\pi}\left[(lnc)z_0 + lnc_1(z_2 - z_0) + lnc_2(z_4 - z_2) + ln\left(\frac{b}{r_p}\right)(z - z_4) + \frac{(z_1 - z_0 + z_3 - z_2)}{2} + \frac{(z_2 - z_1)d_{11}^2}{2} + \frac{(z_4 - z_3)d_{22}^2}{2}\right] \quad 12)$$

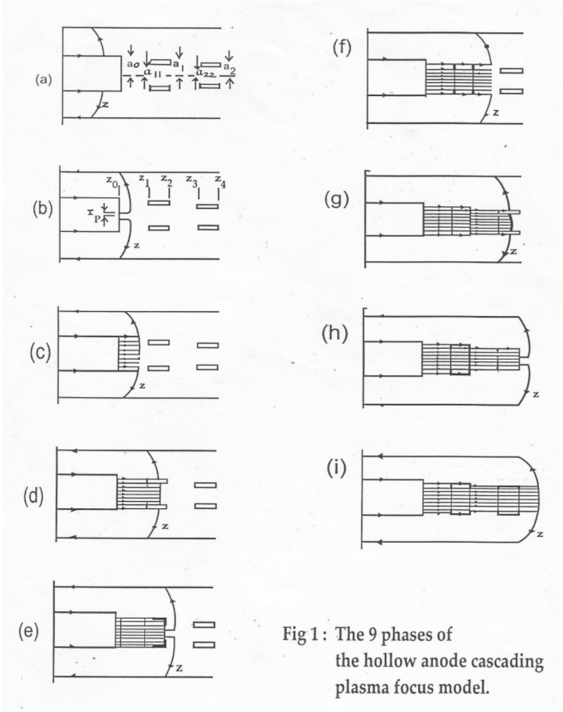

Fig 1 : The 9 phases of the hollow anode cascading plasma focus model.

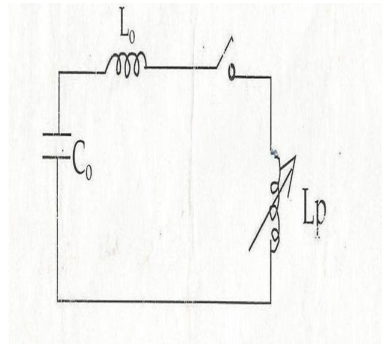

Fig. 2: Equivalent circuit of the cascading plasma focus.

(3) Radial Extension Phases

Inductance in Phase 3 (c in Fig. 1)

$$L_{p3} = \frac{\mu_0}{2\pi}\left[lnc\left(z_0 - z_f\right) + \frac{z_f}{2}\right] \quad 13)$$

Inductance in Phase 6 (f in Fig. 1)

$$L_{p6} = \frac{\mu_0}{2\pi}\left[(lnc)z_0 + lnc_1(z - z_0) + \frac{(z_1 - z_0 + z - z_2)}{2} + \frac{(z_2 - z_1)d_{11}^2}{2}\right] \quad 14)$$

Inductance in Phase 9 (i in Fig. 1)

$$L_{p9} = \left[(lnc)z_0 + lnc_1(z_2 = z_0) + lnc_2(z - z_2) + \frac{(z_1 - z_0 + z_3 - z_2 + z - z_4)}{2} + \frac{(z_2 - z_1)d_{11}^2}{2} + \frac{(z_4 - z_3)d_{22}^2}{2}\right] \quad 15)$$

Mass loading in phase 3

$$m_3 = \pi\rho_0 a^2\left[(c^2 - 1)z_0 + c^2 z_f\right] \quad 16)$$

Mass loading in Phase 6

$$m_6 = \pi\rho_0 a^2[(c^2 - 1)z_0 + (z_2 - z_0)\{c^2 + d_1^2(d_{11}^2 - 1)\} + c^2(z - z_2)] \quad 17)$$

Mass loading in Phase 9

$$m_9 = \pi\rho_0 a^2[(c^2 - 1)z_0 + (z_2 - z_0)\{c^2 + d_1^2(d_{11}^2 - 1)\} + (z_4 - z_2)\{c^2 + d_2^2(d_{22}^2 - 1)\} + c^2(z - z_4)] \quad 18)$$

Axial force in Phase 3

$$F_3 = \frac{\mu_0}{4\pi}\left[lnc + \frac{1}{4}\right]i^2 \quad 19)$$

Axial force in Phase 6

$$F_6 = \frac{\mu_0}{4\pi}\left[lnc_1 + \frac{1}{4}(1 + d_{11}^4)\right]i^2 \qquad 20)$$

Axial force in Phase 9

$$F_9 = \frac{\mu_0}{4\pi}\left[lnc_2 + \frac{1}{4}(1 + d_{22}^4)\right]i^2 \qquad 21)$$

GENERALIZED EQUATIONS I

For each of the 3 axial phases nA (n = 1, 2, 3 successively) equations (1) to (3) can be combined and written as

$$L_{nA} = \mu' \begin{bmatrix} lnc_{n-1}(z - z_{2n-4}) + lnc_{n-2}(z_{2n-4} - z_{2n-6}) + lnc_{n-3}z_{2n-6} + \frac{1}{2}\begin{pmatrix} z_{2n-3} - z_{2n-4} + z_{2n-5} - \\ -z_{2n-6} \end{pmatrix} + \\ + \frac{1}{2}(z - z_{2n-3})d_{(n-1)(n-1)}^2 + \frac{1}{2}(z_{3n-7} - z_{3n-8})d_{(n-2)(n-2)}^2 \end{bmatrix} \qquad 22)$$

where ln0 = 0 in this model, $\mu' = \frac{\mu_0}{2\pi}$, $c_n = \frac{b}{a_n}$, $d_n = \frac{a_n}{a_0}$, $d_{nn} = \frac{a_{nn}}{a_n}$, μ_0 is the permeability of free space. A quantity with single or double negative subscript or double zero subscript has a value zero i.e.$z_{-1} = d_{00} = d_{-1-1} = 0$

Hence the inductance in the axial phase i.e. (a) in Fig.1 put n = 1 in eq.(22) to get L_{1A}. Similarly for second and third axial phases put n = 2 and n = 3 to get L_{2A} and L_{3A} respectively.

The mass loading equations (4) to (6) can be written as

$$m_{nA} = \pi\rho_0 {a_0}^2 \begin{bmatrix} (c_0^2 - d_{n-1}^2)(z - z_{2n-4}) + (c_0^2 - d_{n-2}^2)(z_{2n-4} - z_{2n-6}) + (c_0^2 - d_{n-3}^2)z_{2n-6} + \\ +(z - z_{2n-4})d_{(n-1)}^2 d_{(n-1)(n-1)}^2 + (z_{2n-4} - z_{2n-6})d_{(n-2)}^2 d_{(n-2)(n-2)}^2 \end{bmatrix} \qquad 23)$$

Hence n = 1 gives m_{1A} and n = 2 and 3 gives m_{2A} and m_{3A} respectively.

Similarly the corresponding force for each of the 3 axial phases is

$$F_{nA} = \frac{\mu'}{2}\left[lnc_{n-1} + \frac{d_{(n-1)(n-1)}^4}{4}\right]i^2 \qquad 24)$$

For each of the 3 Radial phases nR (n = 1, 2, 3 successively) eqs. (10) to (12) can be written as

$$L_{nR} = \mu' \begin{bmatrix} ln\frac{b}{r_p}z_f + lnc_{n-1}(z_{2n-2} - z_{2n-4}) + lnc_{n-2}(z_{2n-4} - z_{2n-6}) + lnc_{n-3}z_{2n-6} + \\ + \frac{1}{2}(z_{2n-3} - z_{2n-4} + z_{2n-5} - z_{2n-6}) + \frac{1}{2}(z_{2n-2} - z_{2n-3})d_{(n-1)(n-1)}^2 + \\ + \frac{1}{2}(z_{3n-7} - z_{3n-8})d_{(n-2)(n-2)}^2 \end{bmatrix} \qquad 25)$$

Where $z_f = z - z_{2n-2}$

For each of the 3 Radial Extension phases n C (n=1, 2, 3 successively) eqs. (13) to (15) can be written as

$$L_{nC} = \mu' \begin{bmatrix} lnc_{n-1}(z - z_{2n-4}) + lnc_{n-2}(z_{2n-4} - z_{2n-6}) + lnc_{n-3}z_{2n-6} + \\ + \frac{1}{2}(z - z_{2n-2} + z_{2n-3} - z_{2n-4} + z_{2n-5} - z_{2n-6}) + \frac{1}{2}(z_{2n-2} - z_{2n-3})d_{(n-1)(n-1)}^2 + \\ + \frac{1}{2}(z_{3n-7} - z_{3n-8})d_{(n-2)(n-2)}^2 \end{bmatrix} \qquad 26)$$

In the same vein we write the 3 mass loading in the radial extension phases equations (16), (17) and (18) as

$$m_{nC} = \pi\rho_0 a_0^2 \begin{bmatrix} c_0^2(z - z_{2n-2}) + (c_0^2 - d_{n-1}^2)(z_{2n-2} - z_{2n-4}) + (c_0^2 - d_{n-2}^2)(z_{2n-4} - z_{2n-6}) + \\ +(c_0^2 - d_{n-3}^2)z_{2n-6} + (z_{2n-2} - z_{2n-4})d_{(n-1)}^2 d_{(n-1)(n-1)}^2 + \\ +(z_{2n-4} - z_{2n-6})d_{(n-2)}^2 d_{(n-2)(n-2)}^2 \end{bmatrix} \quad 27)$$

The corresponding force is given by

$$F_{nC} = \frac{\mu'}{2}\left[lnc_{n-1} + \frac{1}{4} + \frac{d_{(n-1)(n-1)}^4}{4}\right] i^2 \quad 28)$$

THE EQUATIONS GOVERNING THE VARIOUS PHASES

A The Axial Phases nA (n=1,2,3 successively)

For the 3 axial phases the governing equations area

(i) Axial motion $\frac{d}{dt}\left(m_{nA}\frac{dz}{dt}\right) = F_{nA}$ 29)

(ii) Circuit Equation $\frac{d}{dt}[(L_0 + L_{nA})i] = V_0 - \frac{\int idt}{C_0}$ 30)

These two coupled equations are solved for z and i step by step starting with t = 0 for the first axial phase n=1 with known conditions.

B The Radial Phases nR (n = 1,2,3 successively)

For each of the radial phases there are 4 governing equations to follow the trajectories of the radial shock front r_s, the radial magnetic piston r_p, increasing length of the pinching column z_f and the current i. The equations for r_s and r_p for a constant current pinch has been discussed by (Potter 1978) this slug model has been modified for an elongation pinch with circuit coupled current to give the following:

(i) Radial shock position

$$\frac{dr_s}{dt} = -\sqrt{\frac{\mu_0}{\rho_0}(\gamma + 1)}\frac{i}{4\pi r_p} \quad 31)$$

(ii) Axial elongation

$$\frac{dz_f}{dt} = -\left(\frac{2}{\gamma+1}\right)\frac{dr_s}{dt} \quad 32)$$

(iii) Radial piston position

$$\frac{dr_p}{dt} = \frac{\frac{2}{\gamma+1}R'\frac{dr_s}{dt} - \frac{r_p}{\gamma i}(1-R'^2)\frac{di}{dt} - \frac{1}{\gamma+1}(1+R'^2)\frac{dz_f}{dt}\frac{r_p}{z_f}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma}R'^2} \quad 33)$$

(iv) Circuit Equation $\frac{d}{dt}[(L_0 + L_{nR})i] = V_0 - \frac{\int idt}{C_0}$ 34)

C The Expanded Column Phases nC (n = 1,2,3 successively)

For each of the 3 expanded phases we write two governing equations as follows

(i) Axial motion $\frac{d}{dt}\left[m_{nC}\frac{dz}{dt}\right] = F_{nC}$ 35)

(ii) Circuit $\frac{d}{dt}[(L_0 + L_{nC})i] = V_0 - \frac{\int idt}{C_0}$ 36)

NORMALIZATION AND INTEGRATION

The equations are normalized with parameters common as follows

(i) Common to all the phases: $\tau = \frac{t}{t_0}$,$\iota = \frac{i}{i_0}$

(ii) For the axial and expanded column phases: $\zeta = \frac{z}{z_0}$

(iii) For the radial phases: $\kappa_s = \frac{r_s}{a_0}$, $\kappa_p = \frac{r_p}{r_0}$, $\zeta_f = \frac{z_f}{a_0}$ where $z_f = z - z_{2n-2}$ for each phase i.e. for 1R (n=1) for 2R (n=2) for 3R (n=3).

GENERALIZED EQUATION II

Thus the normalized equations in generalized form are:

A. Axial phases nA (n = 1,2,3 successively)

(i) Axial motion: $\frac{d^2\zeta}{d\tau^2} = \frac{\alpha^2 f_{n-1}\iota^2 - h_{n-1}\left(\frac{d\zeta}{d\tau}\right)^2}{\zeta_{2n-6}+h_{n-2}(\zeta_{2n-4}-\zeta_{2n-6})+h_{n-1}(\zeta-\zeta_{2n-4})}$ 37)

(ii) Circuit :$\frac{d\iota}{d\tau} = \frac{1-\int \iota d\tau - \beta \bar{f}_{n-1}\tau\frac{d\zeta}{d\tau}}{1+\beta g_{n-2}+\beta g_{2n-5}(\zeta_{2n-4}-\zeta_{2n-5})+\beta(\zeta_{2n-3}-\zeta_{2n-4}+\zeta_{2n-5}-\zeta_{2n-6})+\beta g_{n-1}(\zeta-\zeta_{2n-4})+ +\beta_{2n-5}(\zeta_{2n-4}-\zeta_{2n-5})+\beta_{n-1}(\zeta-\zeta_{2n-3})}$ (38)

Equations (37) and (38) have scaling parameters as **α** and β ,$\alpha^2 = \frac{t_0^2}{t_a^2}$ scales the electrical characteristic time t_0 to the characteristic axial transit time t_a and $\beta = \frac{\mu' \ln c_0 z_0}{L_0}$ which scales the characteristic axial inductance $\mu' \ln c_0 z_0$ to the external circuit inductance L_0.

To start the numerical step by step integration of equations (37) and (38) of the two time varying unknowns ζ and ι we consider the axial phase of the first focus event i.e. phase 1A (n=1) with the starting values as $\tau_0 = 0, (\int \iota d\tau)_0 = 0, \quad \iota_0 = 0, \left(\frac{d\iota}{d\tau}\right)_0 = 1, \zeta_o = 0$, $\left(\frac{d\zeta}{d\tau}\right)_0 = 0, \left(\frac{d^2\zeta}{d\tau^2}\right)_0 = \alpha\sqrt{\frac{2}{3}}$ and a time increment of $\Delta\tau = 0.001$, the simple Euler linear approximation scheme is employed to generate $(\int \iota d\tau)_{n+1}$ and ι_{n+1} from $\left(\frac{d\iota}{d\tau}\right)_n$ using the equations:$(\int \iota d\tau)_{n+1} = (\int \iota d\tau)_n + \iota_n\Delta\tau + \frac{1}{2}\left(\frac{d\iota}{d\tau}\right)_n(\Delta\tau)^2$ and $\iota_{n+1} = \iota_n + \left(\frac{d\iota}{d\tau}\right)_n\Delta\tau$

Similarly ζ_{n+1} and $\left(\frac{d\zeta}{d\tau}\right)_{n+1}$ are generated from $\left(\frac{d^2\zeta}{d\tau^2}\right)_n$ using the equations:

$\zeta_{n+1} = \zeta_n + \left(\frac{d\zeta}{d\tau}\right)_n\Delta\tau + \frac{1}{2}\left(\frac{d^2\zeta}{d\tau^2}\right)_n(\Delta\tau)^2and\left(\frac{d\zeta}{d\tau}\right)_{n+1} = \left(\frac{d\zeta}{d\tau}\right)_n + \left(\frac{d^2\zeta}{d\tau^2}\right)_n\Delta\tau$

From the values of $(\int \iota d\tau)_{n+1}$, ι_{n+1} , ζ_{n+1} and $\left(\frac{d\zeta}{d\tau}\right)_{n+1}$ together with equations (37) and (38) , the next values of the derivatives $\left(\frac{d^2\zeta}{d\tau^2}\right)_{n+1}$ and $\left(\frac{d\iota}{d\tau}\right)_{n+1}$ can be generated. For the axial phases nA (n=1,2,3) the integration is continued until $\zeta = \zeta_{2n-2}$ i.e. for n=1, $\zeta = \zeta_0$ n=2, $\zeta = \zeta_2$ n=3, $\zeta = \zeta_4$ and the starting values of 2A and 3A integration are the following :$\zeta = \zeta_{2n-3}$ and the final values of τ, $\int \iota d\tau$, ι, $\frac{d\iota}{d\tau}$ and $\frac{d^2\zeta}{d\tau^2}$ for the preceding phase.

B Radial Phase nR (n=1,2,3 successively)

The four governing equations in normalized form are:

(1) Radial shock $\frac{d\kappa_s}{d\tau} = -\frac{\alpha\alpha_1\iota}{\kappa_p}$ 39)

(2) Axial elongation $\frac{d\zeta_f}{d\tau} = -\left(\frac{2}{\gamma+1}\right)\frac{d\kappa_s}{d\tau}$ 40)

(3) Radial piston position

$$\frac{d\kappa_p}{d\tau} = \frac{\frac{2}{\gamma+1}R''\frac{d\kappa_s}{d\tau} - \frac{\kappa_p}{\gamma\tau}(1-R''^2)\frac{d\iota}{d\tau} - \frac{1}{(\gamma+1)}\frac{\kappa_p}{\zeta_f}(1-R''^2)\frac{d\zeta_f}{d\tau}}{\left(\frac{\gamma-1}{\gamma}\right)+\frac{R''^2}{\gamma}} \qquad 41)$$

(4) Circuit

$$\frac{d\iota}{d\tau} = \frac{1-\int \iota d\tau + \frac{\beta_f \iota \zeta_f}{\kappa_p}\frac{d\kappa_p}{d\tau} + \beta_f \ln\left(\frac{\kappa_p}{c_0}\right)\iota\frac{d\zeta_f}{d\tau}}{\begin{array}{c}1-\beta_f\left(\ln\frac{\kappa_p}{c_0}\right)\zeta_f+\beta g_{n-2}+\beta g_{2n-5}(\zeta_{2n-4}-\zeta_{2n-6})+\beta g_{n-1}(\zeta_{2n-2}-\zeta_{2n-4})+ \\ +\beta_1(\zeta_{2n-3}-\zeta_{2n-4}+\zeta_{2n-5}-\zeta_{2n-6})+\beta_{n-1}(\zeta_{2n-2}-\zeta_{2n-3})+\beta_{2n-5}(\zeta_{2n-4}-\zeta_{2n-5})\end{array}} \qquad 42)$$

To start the integration of each radial phase we use as starting values the following final values of the corresponding preceding axial phase : τ , $\int \iota d\tau$,$\boldsymbol{\iota}$, $\frac{d\iota}{d\tau}$. Additional initializing values are :$\kappa_s = \kappa_p = d_{n-1}$

$\zeta_f = 0$. To avoid division of zero we use $\zeta_f = 0.00001$.The numerical integration continues for each case until $\kappa_s = 0$ at which point κ_p is minimum.

C The Expanded Column Phase nC (n=1, 2, 3 successively)

The two governing equations in normalized form are :

(1) Axial motion

$$\frac{d^2\zeta}{d\tau^2} = \frac{\alpha^2\iota^2 e_{n-1} - h\left(\frac{d\zeta}{d\tau}\right)^2}{h_{n-3}\zeta_{2n-6}+h_{n-2}(\zeta_{2n-4}-\zeta_{2n-6})+h_{n-1}(\zeta_{2n-2}-\zeta_{2n-4})+h(\zeta-\zeta_{2n-2})} \qquad 43)$$

(2) Circuit Equation

$$\frac{\boldsymbol{d\iota}}{\boldsymbol{d\tau}} = \frac{1-\int \iota d\tau - \beta\iota\left(\frac{d\zeta}{d\tau}\right)e^*_{n-1}}{\begin{array}{c}1+\beta\zeta_{2n-6}+\beta g_{n-2}(\zeta_{2n-4}-\zeta_{2n-6})+\beta g_{n-1}(\zeta-\zeta_{2n-4})+\beta_1(\zeta-\zeta_{2n-2}+\zeta_{2n-3}- \\ -\zeta_{2n-4}+\zeta_{2n-5}-\zeta_{2n-6})+\beta_{n-1}(\zeta_{2n-2}-\zeta_{2n-3})+\beta_{2n-5}(\zeta_{2n-4}-\zeta_{2n-5})\end{array}} \qquad 44)$$

To start the integration of each expanded column phase nC (n = 1,2,3) we use as starting values the following final values of the corresponding preceding radial phases nR (n = 1,2,3) : τ, $\int \iota d\tau$, $\boldsymbol{\iota}$, and $\zeta = 1 + \frac{\zeta_f}{F}$

(Note that here we have renormalized ζ_f in the radial phase ($\zeta_f = \frac{z_f}{a_0}$) to z_0 i.e. $\zeta_f = \frac{z_f}{z_0}$). Also since the dynamics in this phase is similar to the axial phase, the starting values of $\frac{d\zeta}{d\tau}$ is the final value of the corresponding preceding axial phase nA (n=1,2,3). The integration of each expanded column phase proceeds until $\zeta = \zeta_{2n-1}$; n=3 gives ⌘ = ζ_5 which is taken at some convenient point beyond ζ_4 such as

$$\zeta_5 = \zeta_4 + (\zeta_4 - \zeta_3)$$

VOLTAGE EQUATIONS

Apart from current measurement another informative measurement in the plasma focus is the voltage across the input of the focus. Assuming a purely inductive model, this may be computed as $V_p = \frac{d(L_p i)}{dt}$ where L_p in the various phases are given in equ. (22), (25) and (26). The voltage equations written in normalized form as

$\nu = \frac{v_p}{v_0}$ in the 9-phases where v_0 is the initial voltage on the capacitor is as follows

Axial Phase nA (n = 1,2,3 successively)

$$\nu = \beta\frac{d\iota}{d\tau}\left\{g_{n-2} + g_{2n-5}(\zeta_{2n-4} - \zeta_{2n-6}) + g_{n-1}(\zeta - \zeta_{2n-4}) + \frac{1}{2\ln c_0}(\zeta_{2n-3} - \zeta_{2n-4} + \zeta_{2n-5} - \zeta_{2n-6})\right\}$$
$$+\frac{d\iota}{d\tau}\{\beta_{2n-5}(\zeta_{2n-4} - \zeta_{2n-5}) + \beta_{n-1}(\zeta - \zeta_{2n-3})\} + \beta\iota\bar{f}_{n-1}\frac{d\zeta}{d\tau} \quad 45)$$

Radial Phase nR (n = 1,2,3 successively)

$$\nu = \beta\frac{d\iota}{d\tau}\left\{g_{n-2} + g_{2n-5}(\zeta_{2n-4} - \zeta_{2n-6}) + g_{n-1}(\zeta_{2n-2} - \zeta_{2n-4}) + \frac{1}{2\ln c_0}(\zeta_{2n-3} - \zeta_{2n-4} + \zeta_{2n-5} - \zeta_{2n-6})\right\}$$
$$+\frac{d\iota}{d\tau}\left\{\beta_{2n-5}(\zeta_{2n-4} - \zeta_{2n-5}) + \beta_{n-1}(\zeta_{2n-2} - \zeta_{2n-3}) - \beta_f\zeta_f \ln\left(\frac{\kappa_p}{c_0}\right)\right\} - \beta_f\iota\left\{\ln\left(\frac{\kappa_p}{c_0}\right)\frac{d\zeta_f}{d\tau} + \frac{\zeta_f}{\kappa_p}\frac{d\kappa_p}{d\tau}\right\} \quad 46)$$

Expanded Column Phase nC (n=1, 2,3 successively)

$$\nu = \beta\frac{d\iota}{d\tau}\left\{g_{n-2} + g_{2n-5}(\zeta_{2n-4} - \zeta_{2n-6}) + g_{n-1}(\zeta - \zeta_{2n-4}) + \frac{1}{2\ln c_0}(\zeta - \zeta_{2n-2} + \zeta_{2n-3} - \zeta_{2n-4} + \zeta_{2n-5} - \zeta_{2n-6}) + \frac{\beta_{2n-5}}{\beta}(\zeta_{2n-4} - \zeta_{2n-5}) + \frac{\beta_{n-1}}{\beta}(\zeta_{2n-2} - \zeta_{2n-3})\right\} + \beta\iota\frac{d\zeta}{d\tau}\left(g_{n-1} + \frac{1}{2\ln c_0}\right) \quad 47)$$

RESULTS AND DISCUSSIONS

The result of calculation is carried out for the 9-phase hollow anode cascading plasma focus with the equations in generalized form. Since each focus event is made up of three phases and there are three events making up the 9-phases, instead of treating each phase differently where we have about 27 equations, we have reduced it to 11equations as seen from the Generalized Equation II i.e. equ. (37) to (47). The parameters used are the parameters for hollow anode cascade focus i.e. α = 1.7, **β** = 0.27, F = 12.63, c = 3.37,

$\boldsymbol{\gamma}$ =1.667, ζ_1 = 1.083, ζ_2= 1.167, ζ_3 = 1.25, ζ_4= 1.33, d_1=0.895, $d_2 = 0.737$, $d_{11} = 0.765$, $d_{22} = 0.714$, $F_p = 0.32$. These parameters correspond to operation at $V_0 = 15KV$, $C_0 = 30\mu F$, L_0 = 110nH, a = -0.95cm, b = 3.20cm, $z_0 = 12cm$, $z_1 = 13cm$, $z_2 = 14cm$, $z_3 = 15cm$, $z_4 = 16cm$, $a_1 = 0.85cm$, $a_2 = 0.70cm$,

$\boldsymbol{a_{11}} = \mathbf{0.65}\boldsymbol{cm}$, $\boldsymbol{a_{22}} = \mathbf{0.50}\boldsymbol{cm}$ and a pressure of 1torr deuterium. The result is shown in Fig. 3(a) i e. current ($\boldsymbol{\iota}$) with a significant current dip and Fig. 3(b), voltage waveform ($\boldsymbol{\nu}$) with large voltage spike.

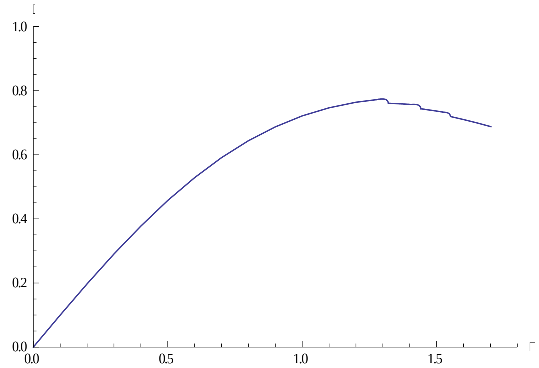

Fig 3a: Current waveform of the hollow anode focus with a = 1.7, b = 0.27, F = 12.63, c = 3.37, g = 1.667, z_1 = 1.083, z_2 = 1.167, z_3 = 1.25, z_4 = 1.33, d_1 = 0.895, d_2 = 0.737, d_{11} = 0.765, d_{22} = 0.714, F_p = 0.32

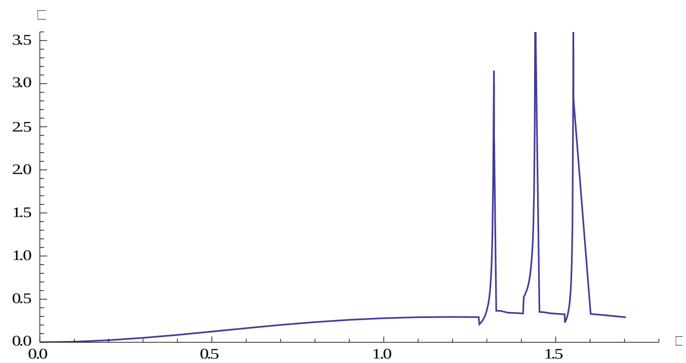

Fig 3b : Voltage waveform of the hollow anode focus with parameters same as in in Fig 3a

CONCLUSION

By the use of Generalized Equations we have been able to reduce the unwieldy equations from 27 to 11.

APPENDIX

γ ratio of specific heat of the plasma $t_a = \sqrt{\frac{4\pi^2\rho_0(c_0^2-1)z_0^2}{\ln c_0\left(\frac{i_0}{a_0}\right)^2}}$ $z_f = z - z_{2n-2}$

$R' = \frac{r_s}{r_p} R'' = \frac{\kappa_s}{\kappa_p} t_0 = \sqrt{L_0 C_0} i_0 = \frac{V_0}{\sqrt{\frac{L_0}{C_0}}}$

$$f_{n-1} = \frac{\ln c_{n-1} + \frac{d^4_{(n-1)(n-1)}}{4}}{\ln c_0} \quad \bar{f}_{n-1} = \frac{\ln c_{n-1} + \frac{d^2_{(n-1)(n-1)}}{2}}{\ln c_0} \quad d_{n-1} = \frac{a_{n-1}}{a_0} \quad d_{n-2} = \frac{a_{n-2}}{a_0}$$

$$h_{n-1} = \frac{c_0^2 + d^2_{n-1}\left(d^2_{(n-1)(n-1)} - 1\right)}{c_0^2 - 1} \quad h_{n-2} = \frac{c_0^2 + d^2_{n-2}\left(d^2_{(n-2)(n-2)} - 1\right)}{c_0^2 - 1} \quad \zeta_{2n-4} = \frac{z_{2n-4}}{z_0}$$

$$\zeta_{2n-6} = \frac{z_{2n-6}}{z_0} \quad h = \frac{c_0^2}{c_0^2 - 1} \quad h_{n-3} = \frac{c_0^2 + d^2_{(n-3)}\left(d^2_{(n-3)(n-3)} - 1\right)}{c_0^2 - 1} \quad e_{n-1} = \frac{\ln c_{n-1} + \frac{1 + d^4_{(n-1)(n-1)}}{4}}{\ln c_0}$$

$$e^*_{n-1} = \frac{\ln c_{n-1} + \frac{1}{2}}{\ln c_0} \quad \mathrm{F} = \frac{z_0}{a_0} \quad \zeta_f = \frac{(z - z_{2n-2})}{z_0} \quad \alpha_1 = \frac{(\gamma+1)\sqrt{(c_0^2 - 1)}\,F}{2\ln c_0} \quad \beta_f = \frac{\beta}{F \ln c_0} \quad \beta_1 = \frac{\beta}{2\ln c_0}$$

$$g_{n-1} = \frac{\ln c_{n-1}}{\ln c_0} \quad g_{n-2} = \frac{\ln c_{n-2}}{\ln c_0} \quad g_{2n-5} = \frac{\ln c_{2n-5}}{\ln c_0} \quad \beta_{n-1} = \frac{\beta d^2_{(n-1)(n-1)}}{2\ln c_0} \quad \beta_{2n-5} = \frac{\beta d^2_{(2n-5)(2n-5)}}{2\ln c_0}$$

Note that when n=3, $g_{n-2} = \frac{\ln c_1}{\ln c_0}$ and from the dimension of the problem $g_{n-2} \cong 1$

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