THE USE OF WEIBULL DISTRIBUTION METHOD FOR ANALYSIS OF WIND SPEED DISTRIBUTION IN KADUNA, NIGERIA.

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ABSTRACT

This paper analyzes the average daily wind speed data obtained from 2004-2014 at the Kaduna Meteorological Station, located at Kaduna Airport. Wind speed were measured at a height of 10m, above ground level using a 3-cup anemometer, at a recording interval of 10min. The wind speed data was analyzed using Weibull distributions in order to investigate the Weibull shape and scale parameters. The Weibull parameters were obtained using the Energy Pattern Factor Method and from these parameters, the Weibull Probability Density Function curve and Weibull Cumulative Distribution Function curve were plotted and interpreted. Results showed that the monthly Weibull shape parameter was in the range of 3.489-7.574m/s, while the monthly Weibull scale parameter was in the range of 3.6837- 7.3304m/s corresponding to monthly mean wind speeds in the range of 4.67-11.79m/s. The monthly power density was in the range of 127.62-400.58 W/m^2 corresponding to the wind power class ranging from class 2-7. Results showed-strong and sufficient winds for power generation during the months of January-September, and December. This implies that the wind speed distribution for Kaduna is very good for wind power generation.

INTRODUCTION

The unsteady nature of wind is a common observation in wind studies; hence, wind speed variations must be properly described and analysed when assessing the Wind Energy Potential of a particular location. Wind turbine designers and engineers need this information in order to optimise the design of their turbines, to minimise generating costs. Also, Wind turbine investors need the information to estimate their income from electricity generation. Since the power curves of Wind Turbine Generators (WTG) are usually made known by manufacturers, the probability distribution of wind speeds is the key information needed to estimate wind energy output at a given site for a particular WTG model (Waewsak *et al,* 2011)

Wind speed variations of a typical location are usually described using Weibull distribution. The Weibull distribution technique has gained a wide acceptance in the wind energy industry as the preferred method for describing wind speed variations at a given site. Justus *et al* (1976) applied the Weibull and Lognormal distribution to Wind speed data from more than a hundred stations of the USA and concluded that Weibull Distribution rendered the best fit. Although, some researchers prefer the Rayleigh distribution, a special case of the Weibull distribution, which sets the shape parameter, k at k = 2; Hennessey (1977) found the Energy output calculated by a Rayleigh distribution to be within 10% of the output based on Weibull distribution.

WEIBULL DISTRIBUTION

The Weibull distribution is a continuous probability distribution named after the Swedish Engineer, Physicist and Mathematician Waloddi Weibull, who popularized its use for reliability analysis, when he applied it when studying materials in tension and fatigue. Although Weibull's paper on the subject was first published in 1939, the method did not attract much

attention, until the 1950's. The Weibull distribution method provides a close approximation to the probability laws of many natural phenomena, and has been used to represent wind speed distribution for application in wind load studies for some time. A lot of attention has been focused on this method for wind energy application in recent years, not only due to its greater flexibility and simplicity but also because it can give a good fit to experimental data (Waewsak *et al*, 2011)
Its Probability Density Function (PDF) is mathematically expressed for wind speed as:

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (1)$$

And its Cumulative Distribution Function (CDF) expressed as:

$$F(v) = 1 - exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (2)$$

Where $k > 0$ is the *shape parameter* and $c > 0$ is the *scale parameter* of the distribution, v is wind speed measured at different intervals in a day.
Ohunakin *et al* (2012) used Weibull Distribution method for wind evaluation of some locations in Nigeria, and showed that the monthly Weibull shape and scale parameters at a height of 10m for Kaduna was in the range of 3.489- 7.574 m/s and 3.6837- 7.3304 m/s respectively. Wind speed is dynamic and changes over time; hence, it is necessary to update these researches overtime to ascertain the state of the climatic condition (speed) and compare with previous researches. Secondly, this paper uses Weibull analysis to estimate the average monthly and annual wind most probable velocities (v_{mp}), the average monthly and annual wind maximum energetic velocities ($v_{\max E}$) and the average monthly and annual available wind power densities(P_d), hence, the justification for this work.

Estimation of Weibull Shape and Scale Parameters
There are several methods for estimating the Weibull shape and scale parameters, some of which include: The Empirical Method (a special case of the Moment Method, the Maximum Likelihood Method, the Modified Maximum Likelihood Method, and the Energy Pattern Factor (EPF) Method.

The energy pattern factor method is a new, simpler formulation, easier implementation, and less computation method suggested by Akdag and Ali (2009). It was described by Indhumathy *et al* (2014) as best fit for estimating Weibull parameters; hence, it is adopted in this work.

Energy Pattern Factor (EPF) Method
The EPF is defined by the following equation:

$$EPF = \frac{1}{(\bar{v})^3} \times \left(\frac{\sum_{i=1}^{n} v_i{}^3}{n}\right) \quad (3)$$

where v_i is the wind speed in meter per second for the ith observation, n is the number of wind speed samples, and $\bar{v}$ is the monthly mean wind speed.
The Weibull shape and scale parameters is derived from the EPF by Indhumathy *et al* (2014) as:

$$k = 1 + \frac{3.69}{(EPF)^2} \quad (4)$$

$$c = \frac{\bar{v}}{\Gamma(1 + \frac{1}{k})} \quad (5)$$

Where Γ is the gamma function defined in general x-variable (Odo et al, 2013) as:

$$\Gamma(x) = \int_0^\infty x^{n-1} e^{-x} dx \quad (6)$$

METHODOLOGY

Average daily wind speed data from 2004-2014 was obtained from the Nigerian Meteorological (NIMET) stations located in the Kaduna Airport, Mando, Kaduna. The sites coordinates are: Longitude 07^{o}19$^{'}$E and Latitude-10^{o}41$^{'}$N. The Altitude of the site is 632m above the sea level.

Instrumentation and Data Acquisition

The average daily wind speed data was captured by a cup-anemometer (Figure 1) at a height of 10m above ground level, and was converted from knots to m/s. The cup-anemometer consists of three cups attached to short rods that are connected to a vertical shaft at right angles. When the wind blows, it pushes the cups, which turn the shaft. (Encyclopædia Britanica, 2012). The number of turns per minute is translated into wind speed by a system of gears similar to the speedometer of an automobile (Redmond, 2008)

$$1\ turn = 2\pi r \tag{7}$$

Where r is the length of one arm of the anemometer.

Figure 1: Cup-Anemometer

The cubed of daily wind speed for each day was computed. And the monthly average for an eleven year period was found as shown in Table 1. The cubed of the monthly and annual mean speeds were also computed as shown in Table 2. The monthly and annual energy pattern factors were then computed using equation 3 as shown in Table 3. The Weibull shape and scale parameters were then computed using equations 4 and 5 respectively (Table 3), to be used for the wind speed distribution analysis.

Wind Speed Distribution Analysis

The Weibull shape and scale parameters, k and c, corresponding to the wind speed distributions measured at 10m above ground level, were used in analysis of the most probable velocities, v_{mp} (also called the most frequent wind speed), and the maximum energetic wind speed, $v_{\max E}$ by using the correlations expressed in equations (8) and (9) respectively.

$$v_{mp} = c\left(\frac{k-1}{k}\right)^{1/k} \tag{8}$$

$$v_{\max E} = c\left(\frac{k+2}{k}\right)^{1/k} \tag{9}$$

Furthermore, the average monthly and annual wind power densities, P_d were calculated in terms of the Weibull Probability Density Function, using the correlation in equation (10)

$$P_d = \frac{1}{2}\rho \bar{v}^3\ X\ (PDF) \tag{10}$$

Where ρ is the air density (kg/m^3), (Table 3)

Finally, the Weibull Probability density Function curves (Figures 2 and 3) and the Weibull Cumulative Distribution Function curves were plotted (Figures 4 and 5) and the Wind Energy Potential of Kaduna was assessed using the 10m above ground level table Wind power class chart, Table 4..

Table 1: Eleven Year Period (2004-2014) Computed Monthly Average Wind Speed in m/s for Kaduna Town

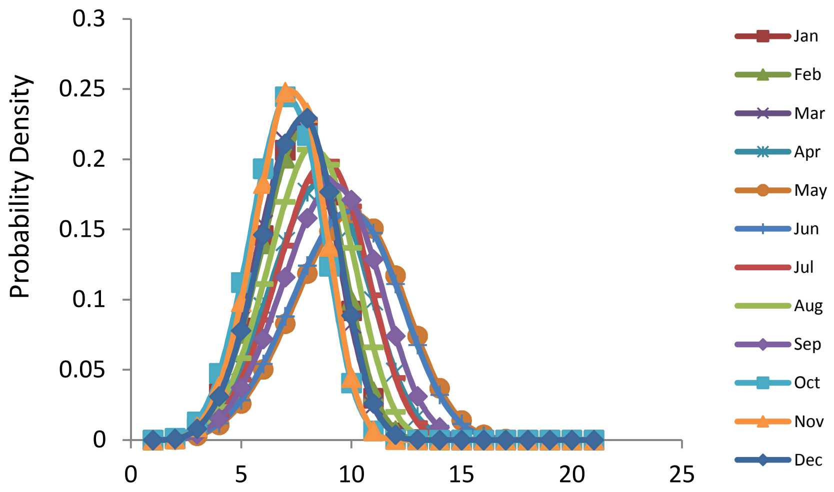
Year/Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Mean
2004	5.64	8.81	6.75	6.40	8.19	8.67	7.99	5.19	7.78	4.89	5.57	4.67	6.71
2005	6.33	6.21	5.45	6.30	7.35	6.00	6.33	7.39	6.66	5.53	6.56	5.81	6.33
2006	5.60	4.92	5.48	6.94	9.34	8.52	6.02	6.29	7.99	6.48	6.57	6.75	6.74
2007	7.43	6.32	5.08	7.26	8.09	9.04	6.38	6.66	5.87	5.66	5.22	6.74	6.65
2008	7.32	7.36	5.59	6.09	9.34	7.86	7.71	7.40	7.83	6.26	6.20	5.97	7.08
2009	5.35	6.06	6.55	6.93	6.79	8.47	7.79	7.04	6.63	4.92	5.57	6.27	6.53
2010	5.71	6.10	8.30	7.96	8.82	7.05	7.41	6.41	8.07	5.85	6.33	7.51	7.13
2011	8.37	6.53	7.66	8.99	11.79	8.17	9.18	7.91	10.20	7.56	6.71	7.54	8.38
2012	7.44	6.67	7.02	11.03	9.96	9.08	8.36	9.33	8.30	7.44	6.16	6.66	8.12
2013	6.30	6.82	6.73	7.28	10.23	10.49	7.26	6.94	9.70	5.81	5.63	6.69	7.49
2014	6.59	7.13	6.31	7.02	6.95	11.91	8.69	7.16	8.65	5.62	7.03	7.40	7.54
Average Wind Speed	6.55	6.63	6.45	7.47	8.80	8.66	7.56	7.07	7.97	6.00	6.14	6.55	7.15

Table 2: Eleven Year Period (2004-2014) Monthly Cubed Average Wind Speed in m^3/s^3 for Kaduna Town

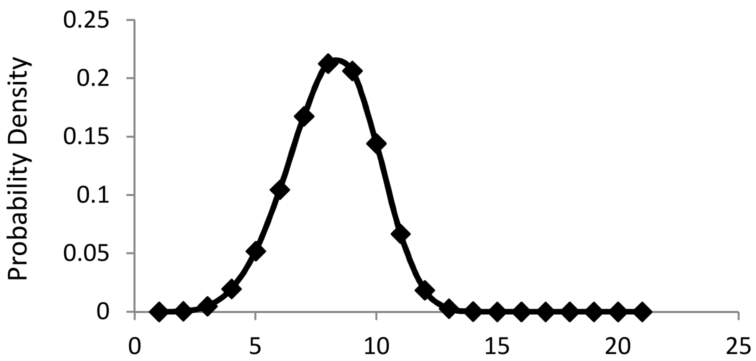
Year/Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Mean (cubed)
2004	179.41	683.80	307.55	262.14	549.35	651.71	510.08	139.80	470.91	116.93	172.81	101.85	302.11
2005	253.64	239.48	161.88	250.05	397.07	216.00	253.64	403.58	295.41	169.11	282.30	196.12	253.64
2006	175.62	119.10	164.57	334.26	814.78	618.47	218.17	248.86	510.08	272.10	283.59	307.55	306.18
2007	410.17	252.44	131.10	382.66	529.48	738.76	259.69	295.41	202.26	181.32	142.24	306.18	294.08
2008	392.22	398.69	174.68	225.87	814.78	485.59	458.31	405.22	480.05	245.31	238.33	212.78	354.89
2009	153.13	222.55	281.01	332.81	313.05	607.65	472.73	348.91	291.43	119.10	172.81	246.49	278.45
2010	186.17	226.98	571.79	504.36	686.13	350.40	406.87	263.37	525.56	200.20	253.64	423.56	362.47
2011	586.38	278.45	449.46	726.57	1638.86	545.34	773.62	494.91	1061.21	432.08	302.11	428.66	588.48
2012	411.83	296.74	345.95	1341.92	988.05	748.61	584.28	812.17	571.79	411.83	233.74	295.41	535.39
2013	250.05	317.21	304.82	385.83	1070.60	1154.32	382.66	334.26	912.67	196.12	178.45	299.42	420.19
2014	286.19	362.47	251.24	345.95	335.70	1689.41	656.23	367.06	647.21	177.50	347.43	405.22	428.66
Average Wind Speed (cubed)	298.62	308.90	285.82	462.95	739.80	709.66	452.39	373.99	542.60	229.24	237.04	293.02	374.96

Table 3: Energy Pattern Factor (EPF), Weibull Shape and Scale Parameters (K and C), Most Probable Velocity (V_{mp}), Maximum Energetic Velocity (V_{mp}), Air Density, Power Density (P_d), Wind Speed For Zaria (2004-2014).

	EPF	k (M/S)	c (M/S)	v_{mp} (M/S)	$v_{max\,E}$ (M/S)	Air Density (Kg/M^3)	P_d (W/M^2)
Jan	1.06	4.27	7.20	6.76	7.88	1.18	166.51
Feb	1.06	4.28	7.29	6.85	7.97	1.18	171.69
Mar	1.07	4.25	7.09	6.66	7.76	1.17	157.16
Apr	1.11	3.99	8.24	7.67	9.12	1.17	243.88
May	1.09	4.13	9.69	9.06	10.66	1.18	400.58
Jun	1.09	4.09	9.54	8.91	10.52	1.18	383.86
Jul	1.05	4.37	8.30	7.82	9.05	1.19	256.26
Aug	1.06	4.30	7.77	7.31	8.49	1.19	209.78
Sep	1.07	4.21	8.77	8.22	9.62	1.18	299.11
Oct	1.06	4.28	6.59	6.19	7.21	1.18	127.62
Nov	1.02	4.52	6.72	6.36	7.29	1.18	136.98
Dec	1.04	4.39	7.18	6.77	7.82	1.19	166.70
Annual average	1.03	4.51	7.83	7.41	8.49	1.18	246.69



Wind Speed (m/s)
Figure 2: Monthly Weibull Distributions (2004-2014) at 10m above ground level for Kaduna.



Wind Speed (m/s)
Figure 3: Average Annual Weibull Distributions (2004-2014) at 10m above ground level for Kaduna.

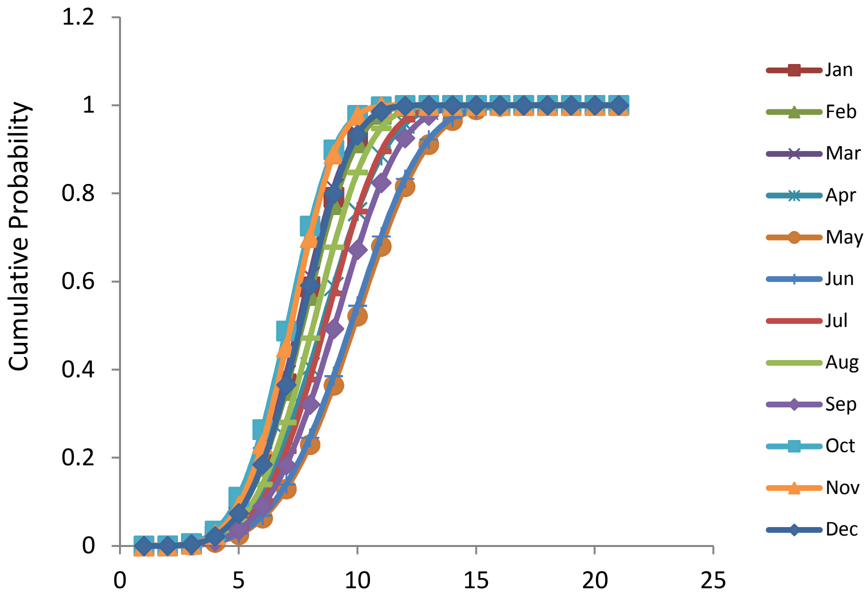


Figure 4: Monthly Weibull Cumulative Probability (2004-2014) at 10m above ground level for Kaduna.

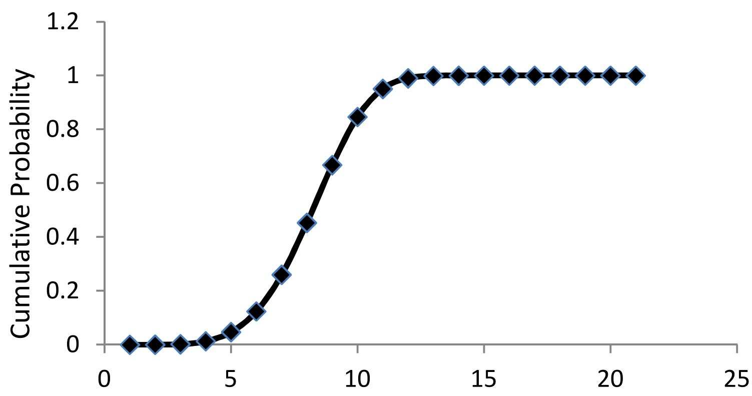


Figure 5: Average Annual Weibull Cumulative Probability (2004-2014) at 10m above ground level for Kaduna.

Table 4: Wind Power Class at 10m Elevation Chart (NREL,2006)

Power Class	Available Power Density (W/m^2)
1	0-100
2	101-150
3	151-200
4	201-250
5	251-300
6	301-400
7	401-1000

Power Class 1 is classified as ‘very low’ (not profitable for wind power generation), Power Class 2 is classified as ‘sufficient to drive small wind turbines’, and Classes 3-7 are classified as able to drive utility sized wind turbines.

RESULTS AND DISCUSSION

Analysis of daily average wind speed data (2004-2014) showed that the month of May had the highest average wind speed at a height of 10m of 8.80m/s; next to it was the month of June with 8.60m/s. The month of October recorded the lowest average wind speed with 6.00m/s. The average annual wind speed for the 11 years was 7.15m/s. The Weibull shape parameters, k ranged from 3.99-4.52m/s, (Table 3) and the Weibull scale parameters, c ranged from 6.59-9.69m/s, (Table 3). The annual average shape and scale parameters were 4.51m/s and 7.83m/s respectively. The Weibull annual average probability distribution curve (Figure 3) showed that the median of the distribution was 8.5m/s, which means that half of the time the wind blows less than 8.5m/s and the other half, the wind blows faster than 8.5m/s. Using equation (8) the estimated monthly most probable velocities, v_{mp} ranged from 6.19-9.06m/s, and using equation (9), the monthly maximum energetic wind speed, v_{maxE} ranged from 7.21-10.66m/s. The cut-in speed for most utility sized turbines is about 5m/s and the cut out speed is approximately 25m/s (NREL, 2006). This implies that the wind speed distribution for Kaduna is very good for wind power generation.

Furthermore the annual available average power density estimated from equation (10) was 246.69 W/m^2. The monthly available wind power densities ranged from 127.62-400.58W/m^2, with the month of May as the highest and the month of October as the lowest. Based on the National Renewable Energy Laboratory (NREL), U.S. Department of Energy (DOE) Wind Power Class Chart, (Table 4), the estimated annual available average power density of 246.69W/m^2 falls in Power Class 4, which is sufficient to power utility sized wind turbines. Also, excess wind energy generated by wind turbines during the months of May and June, can be stored to be used in the months of October and November. Hence, wind energy can be exploited for power generation in Kaduna at a commercial scale, throughout the year.

CONCLUSION AND RECOMMENDATION

The wind distribution for Kaduna was analysed using Weibull analysis. The estimated mean annual wind power density was 246.69 W/m^2, which falls in Wind power Class 4 and is adequate for powering utility sized wind turbines. The Weibull probability density function curve also showed a good distribution of wind speed throughout the year for wind power generation. Although, the estimated average wind power densities for the months of October and November did not meet up to NREL requirements for powering utility sized turbines; the wind power densities of May and June showed sufficient wind power that can be stored for power generation

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